Abstract. This article develops a socio-economic model to investigate the doping decision of professional athletes. In their evaluation of whether to use performance enhancing drugs athletes consider not only costs and benefits (through rank improvement) but also their fellow athletes’ approval for a pro-doping decision. Peer-group approval is modeled as a lagged endogenous variable depending on the share of doping athletes in the history of a sport. This way, the model can explain an equilibrium of high incidence of doping as a “doping culture”. Besides the comparative statics of the equilibrium (how can a doping culture be eliminated?) the article also investigates how the doping decision is affected by standards set by the respective leader in a sport, e.g. Olympic qualification marks, and by the disproportionate public veneration of winners.

Keywords: sport, doping, approval, social dynamics, rank loss aversion, taste for victory, superheroes.

I never cheated.
(Jan Ullrich)

So I did it, but I didn’t feel totally guilty about it because everybody else seemed to be doing it.
(Frankie Andreu)

1. Introduction

Convicted drug users in sports have definitely violated the official rules. Surprisingly often, however, they nevertheless maintain that they were not doing anything wrong and justify this view by mentioning that their fellow athletes are using drugs as well. Apparently, the community of professional athletes sustains a different norm about doping than society at large, a fact, which is addressed by commentators as a doping culture.

In the following I provide a brief introduction of how doping behavior and the notion of a doping culture is conceptualized in the scientific literature on the sociology and psychology of sport. In the main text I then try to translate these concepts into economic language and set up a model to explain the evolution and stability of a doping culture in economic terms. I then use the model to investigate the effectiveness of anti-doping policies and the impact of rules (qualifications marks) and general norms (veneration of winners) on the individual doping decision and on the resulting doping equilibrium assumed by an athletes’ community. For illustration I frequently refer to cycling and the Tour de France but I hope that the theory is general enough to cover also many other sports.\(^1\)

Doping has a long history in many sports but it has been particularly prevalent in cycling. At the time of the first Olympics when Baron de Coubertin promoted the motto that competing is more important than winning, cyclists used wine, cocaine, strychnine, ephedrine, and cocoa leaves in order to enhance their performance (de Rose, 2007). Since then the performance enhancing power of drugs has been constantly on the rise. For example, injections of EPO, available since the early nineties, provide enhancements in endurance performance by about 5\% (Sawka et al., 1996, Birkeland, et al., 2000). This value just exceeds the average gap between the winner’s time and that of the last-place finisher in the last 10 Tours de France, which was 4.75\% (Lindsay, 2007).\(^2\)


\(^2\) Cycling is the sport with the highest percentage of adverse findings (3.78\% of all samples). Second place in this statistic is baseball (3.60\%), then comes boxing and triathlon. Both, de Rose (2007), from which this information was taken, and Catlin et al. (2008) provide excellent overviews of the history of doping from medical perspective. According to the latest available statistic (WADA, 2009a) the international cycling organization UCI reported 45 doping rule violations over the last year, followed by the international swimming organization which reported 26 cases.
The popularity and power of the applied doping method varies of course with the requirements of the sport. While endurance athletes most frequently use methods to increase the oxygen-carrying capacity of blood (EPO, blood doping), Power athletes like sprinters, boxers, and weightlifters, prefer anabolic steroids, and athletes for whom steady action is most important (archers and shooters) prefer sedatives.3

The literature offers two complementing explanations for why doping is so widespread in cycling compared to other endurance sports (for which presumably the same power of drugs is available). First, cycling, in particular the stage races, is frequently considered to be the hardest sport, a fact that makes it easier to develop individually a pro-doping attitude (Waddington, 2000; Mignon (2003); Kimmage, 2006). Second, cycling is a team-sport, a fact that makes it easier to develop and sustain a pro-doping attitude within the group of professional athletes. In the following I develop this argument in more detail.

In many countries it is illegal to prescribe and sell performance enhancing drugs but doping itself is not against the law. Doping athletes must thus not feel guilty from a legal perspective. Doping is forbidden by rules of many athletes’ organizations and the World-anti-doping-agency (WADA). The WADA defines doping as the “presence of prohibited substances in an athlete’s body” and in order to make this abstract definition manageable it has drawn up a so-called negative list of substances.4

If the use of drugs on the list is detected, the deviant athlete is punished with a temporary ban from competition (of two years in many sports). This means that athletes using substances that are not or not yet on the negative list must not feel guilty for violating the rules of their sport. However, irrespective of whether the drug they use is forbidden or not-yet-forbidden, athletes may feel ashamed for cheating on their competitors.

This article argues that the awareness of doping fellow athletes affects the individual doping decision not only through its anticipated effect on rank in competition but also through learning and encouragement from peers (Sutherland and Cressy, 1974). If sufficiently many athletes are doping, the use of drugs may become a norm. Of course, such a doping norm is, if it exits, only operative within the athletes' community. A doping norm, or doping culture, among professional athletes of a sport can be perfectly compatible with a different and probably directly opposed norm of the

3 See Catlin et al. (2008) for a detailed description of composition, effects, and side effects of EPO and other performance enhancing drugs.
4 In this article “drug use” is meant as a convenient simplification of “forbidden performance enhancing behavior” and may thus be thought of encompassing activities like blood doping that involve strictly speaking no intake of drugs. The WADA also bans some recreational, non-performance enhancing drugs (like cannabis) if they are “harmful to the health of the athlete” and “against the spirit of the game”. The non-performance enhancing aspects of drug intake are ignored in this article.
dominant culture, i.e. the general public, the spectators, and the journalists. The dominant culture may, in fact, despise “dirty athletes.”

The degree to which professional athletes form closed communities influences how easily they reject society’s hostile attitude against doping and produce and sustain their own norms. In the case of cycling, for example, Wieting (2000) has observed that “there are two (rather than one) normative frameworks: one of the racers themselves and the other of the surrounding society.” This way, drug use can become widely approved within the group of athletes until it is seen as an essential prerequisite for success. This notion is taken one step further by Coakley and Hughes (1998). They argue that drug use by professional athletes should not be conceptualized as negative deviance but as positive deviance. It expresses an overconformity to key values in sport, most notably the value attached to winning. With respect to professional cycling Mignon (2003) argues that it is not only the desire to win (which is anyway a reasonable goal for only a subgroup of athletes) but also the desire to stay in the game. An athletes’ community has to defend its own norms because “outsiders do not know how hard a rider has to work just to stay in the race”. In other words, the social practice of a sport defines what is conceptualized as “fair play” by the participants (Loland and McNamee, 2000).

There exists some recent empirical research supporting this assessment. Petroczi (2007) analyzes interviews on the doping attitudes and behavior of male college athletes. Generally she finds that doping athletes acknowledge their rule breaking behavior but do not consider themselves as cheaters or more cheating than others. Lentillon-Kaestner and Carstairs (2009) analyze interviews of young elite cyclists and conclude that these athletes believe doping to be acceptable at the professional level (but not on the amateur level), that cyclists who recently became professional experienced pressure from teammates to start doping, and that “more experienced cyclists transmitted the culture of doping to the young cyclists: they gave information about which substance to use and taught the young cyclists the methods”. Implicitly the WADA and many sport organizations have recently acknowledged the power of peer group approval and group cohesion by launching educational programmes designed to reduce this influence, e.g. the “Play True Generation” program (WADA, 2009b) and the “True Champion or Cheat” program of the UCI (2009).

So far, a small economic literature has tried to rationalize the use of drugs in sports, mostly focussing on two players competing in a game-theoretic situation with only two outcomes: winner or

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5The term “doping culture” initially coined by the news press appears to be indeed quite appropriately chosen against the background of thorough definitions of “culture” in sociology and anthropology. A sports community produces its own rules, own language, a code of practice, which is considered to be “normal”, it shares a common code of honor as well as common symbols (the gold medal, the yellow jersey, the sweeper bus).
The present article adds at least two new aspects to this literature: It considers the behavior of many athletes competing about the ranking in their sport and it investigates the role of socially dependent preferences. The focus on ranking instead of winning allows to analyze a richer set of motives for participating professionally in a sport. Many athletes are obviously getting something (utility, prestige, money) out of their rank without managing to be the number one in their sport. While there may be individual contests between just two athletes, the ultimate goal of each athlete is not to win one particular match or tournament but to appear high in the (world) ranking.

The model presented below captures these ideas by assuming that utility of competing athletes depends positively on rank and positively (negatively) on (dis-) approval of a pro-doping decision. Athletes differ with respect to ability and with respect to their susceptibility to approval of their behavior. Peer group approval in turn is derived from the doping history of a sport, i.e. from the number of drug using athletes in the past. Depending on the power of drugs, peer group cohesion, and the monetary and stigma costs of using drugs, the model is capable of generating different equilibria. In particular it can motivate an equilibrium of high incidence of doping which is assumed and sustained “only” because peer-group approval matters for utility and group cohesion is sufficiently strong. After working out these doping equilibria the article continues with an assessment of anti-doping policies and an investigation of some plausible extensions of the basic framework.

2. The Basic Model

Suppose ability of competitors in a sport is uniformly distributed in the unit interval. Athlete $i$ has ability $A(i) \in [0, 1]$. An $A(i)$ of 1 indicates highest ability and an $A(i)$ of 0 lowest ability. Suppose that the rank of athletes in competition, for example at the PGA if the sport is golf or at the Tour de France if the sport is cycling, is uniformly and continuously distributed. In an ideal world ability would map one-to-one into rank, i.e. $R(i) = A(i)$. Yet, there is the possibility of doping to improve rank. Doping is a binary choice, $d \in \{0, 1\}$. An athlete either uses performance enhancing drugs, here denoted by $d = 1$, or he stays clean, denoted by $d = 0$.

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6 Berentsen (2002), Berentsen and Lengwiler (2004), Haugen (2004), and Kräkel (2007). Berentsen et al. (2008) discuss leniency clauses within this framework. Some of the ideas developed in the present article were already mentioned in Bird and Wagner (1997) but they were not formally investigated. Dilger et al. (2007) provide a survey over the theoretical literature in economics and some (sparse) empirical evidence.

7 There exists a small economic literature that uses similar ideas to investigate the welfare state (Lindbeck, et al., 1999), out-of-wedlock childbearing (Nechyba, 2001), occupational choice (Mani and Mullin, 2004) and school attendance and child labor (Strulik, 2010).

8 We ignore the possibility of doping in order to manipulate a competitor’s rank negatively (poisoning), a possibility which is mainly popular in horse racing. Doping a competitor’s horse is excluded from analysis because this behavior is certainly regarded as cheating by the riders’ community and the present article focusses on behavior that is not necessarily regarded as “against the rules” by the athletes, i.e. behavior with potential for peer-group approval.
Given the possibility of doping, an athlete’s rank depends not only on ability alone anymore but also on his doping decision and on the doping decision of his competitors. Let $\theta_t$ denote the share of competitors who are using drugs in season $t$ and let rank of athlete $i$ in season $t$, $R_t(i, d)$, be determined by the following function.

$$R_t(i, d) = A(i) \cdot \{1 + \alpha \cdot [d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t]\}.$$  

According to (1) there are two situations in which rank equals ability. The first one is clean sports: athlete $i$ stays clean and so do all other athletes, $d = \theta = 0$. The other possibility is completely dirty sports: athlete $i$ uses drugs and so do all others athletes, $d = \theta = 1$. In between these borders an athlete can improve his rank through drug intake. This is captured by the first term in brackets. Yet, frequently professional athletes describe their decision on doping with a different twist. They emphasize that they would lose ranking-wise if they opted against doping because (many of) their competitors are using drugs. This side of the argument is captured by the second term in brackets. The loss of rank of a clean athlete is increasing in the share of doping athletes in a sport.\(^9\)

The parameter $\alpha \geq 0$ measures the power of drugs. In the limit $\alpha = 0$ and using drugs has no effect on rank. The value of $\alpha$ characterizes the sport under investigation, which may range from golf, which is generally believed to have a low incidence of doping, to cycling, which is possibly associated with the highest value of $\alpha$.

There are also costs of using drugs. For the basic model, we subsume individual costs in a single parameter, $c$. These costs include the actual monetary costs of the drugs (which athletes frequently pay out of their personal budget) and the expected costs of losing one’s job if being detected as a doper (losing the licence to compete in tournaments for a while, losing attractive advertising contracts). Costs may also include the fear of consequences later in life, like illness and premature death, caused by the unhealthy use of chemical substances.

In bearing these costs it helps if an athlete experiences approval of a pro-doping decision by his peers. The level of peer-group approval in season $t$ is denoted by $S_t \in [0, 1]$. While the level of $S_t$ is given to the individual athlete it is on the aggregate endogenously determined by the share of athletes who were doping in the past. The degree to which athletes are influenced by approval of their actions is individual-specific, also assumed to be uniformly distributed within the unit interval,\(^9\)

\(^9\) Note that $R_t(i)$ is not generally (for all $\alpha$) bounded from above by unity. If rank of an athlete $i$ is conceptualized as the measure of athletes who are ranked lower than $i$, then it is more appropriate to think of $R_t(i)$ as the performance outcome on which the actual ranking is based. Having this qualification in mind we will, for simplicity, refer to $R_t(i)$ as rank of athlete $i$. 

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and denoted by $\sigma(i) \in [0, 1]$. At the highest level of susceptibility $\sigma(i) = 1$ and at the lowest level $\sigma(i) = 0$.

Athletes also experience disapproval of doping, in particular from spectators, the press, and the society at large. Let the marginal strength of disapproval be denoted by $\phi$ so that $\sigma(i) \cdot \phi$ reflects the “stigma costs” experienced by athlete $i$ if he is the one and only in his sport who uses drugs.

The relative importance of approval experienced from peers (compared to general disapproval) is measured by the parameter $\beta > 0$. This parameter tries to measure the cohesion and closeness of an athletes’ community. The strength of community cohesion is potentially sport-dependent, varying, for example, between team sports (as for example cycling) where teammates are more likely to rely on each other and individual sports (track and field) where athletes are potentially less close to each other. Group cohesion is also potentially situation specific (for example, cycling before and after the Festina scandal) and policy dependent (for example, affected by leniency policies for doping convicts).

In sum, social approval of doping – which may turn to disapproval if negative – experienced by athlete $i$ in season $t$ is given by $\sigma(i) \cdot (\beta S_t - \phi)$. Since the maximum peer-group approval is unity, obtained when all athletes are using drugs, we assume that $\beta > \phi$ in order to allow peer-group approval to have a positive influence on doping. Sometimes we will also consider a reference case in which social influences play no role at all, i.e. $\sigma(i) = 0$ for all $i$. Comparing with the case of $\sigma(i) > 0$ we can assess the role of peer-group approval in sports and explain the phenomenon of a doping culture.

Choosing $d \in \{0, 1\}$ athlete $i$ maximizes his net utility consisting of rank minus costs of doping plus utility obtained from social approval. Athlete $i$ in season $t$ maximizes

$$U(i, d) = A(i) \cdot \left\{1 + \alpha \left[d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t\right]\right\} - d \cdot c + d \cdot \sigma(i) \cdot (\beta S_t - \phi), \quad \beta > \phi. \quad (2)$$

If athlete $i$ decides to stay clean and chooses $d = 0$ he receives utility $A(i) (1 - \alpha \cdot \theta_t)$. Utility of a clean athlete is increasing in individual ability and decreasing in the share of doping fellow athletes whereby the magnitude of the loss depends on $\alpha$, the power of drugs. If athlete $i$ decides to use

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10 All arguments made in this article are generally independent from the assumptions about distribution functions and many would be re-enforced by bell-shaped distribution functions. A uniform distribution, however, allows for analytical solution and diagrammatic exposition of equilibria.

11 The Festina scandal, i.e. the doping cases at and around the 1998 Tour France, is generally regarded as the most significant doping affair in sports because it revealed for the first time that an entire athlete community (including trainers, doctors, and officials) were practicing and/or concealing doping (see Waddington (2000), Houlihan (2002) and Mignon (2003)). It has led to the foundation of the WADA and the introduction of much stronger doping controls and punishment. Moreover, Lentillon-Kaestner and Carstairs (2009) provide evidence that it has also changed group cohesion. Before the Festina scandal doping was often organized at the team level. After Festina, doping became a more private and clandestine activity.
drugs, i.e. \( d = 1 \) he receives utility \( A(i) \cdot [1 + \alpha \cdot (1 - \theta_t)] - c + \sigma(i) \cdot (\beta S_t - \phi) \).

Comparing utilities, an athlete decides to stay clean if \( \alpha A(i) \leq c - \sigma(i) (\beta S_t - \phi) \). From this we get the threshold between clean and doping athletes in the two-dimensional ability-susceptibility space:

\[
A = \frac{c - \sigma(\beta S_t - \phi)}{\alpha}.
\] (3)

To begin with consider the special case where social approval plays no role, i.e. \( \sigma(i) = 0 \) for all \( i \). In this case, all athletes with ability above \( c/\alpha \) use drugs. Athletes of high ability are more inclined to dope because, compared to costs, they get more out of performance enhancing drugs in terms of rank improvement and utility.

Turning towards the case of socially-dependent preferences, we make the following assumption.

**Assumption 1.** \( c < \alpha \).

We thus focus on cases where, absent social influences, at least one athlete would have an incentive to use performance enhancing drugs and elaborate how the existence of social interdependence aggravates (or perhaps attenuates) the incidence of doping in a sport.

Figure 1: The Threshold between Clean and Drug-Using Athletes

Athletes are distinguished by ability \( A \in [0, 1] \) and by susceptibility to peer-group approval \( \sigma \in [0, 1] \). The share of drug using athletes in season \( t \) is \( \theta_t \). The strength of peer approval is \( S_t \). If \( S_t > S_{\text{high}} \equiv (\phi + c)/\beta \), there exists a critical \( \bar{\sigma} \) above which all athletes dope. If \( S_t < S_{\text{low}} \equiv (\phi + c - \alpha)/\beta \), there exists a critical \( \bar{\sigma} \), above which all athletes stay clean. For intermediate values of \( S_t \) there exist some doping athletes at any level of \( \sigma \).

Figure 1 displays the three qualitatively distinct situations under socially-dependent preferences. The threshold (3) is always represented by a bold line. A \((\sigma, A)\)-tuple above the threshold identifies a doping athlete. Allowing for social preferences does thus not change the observation that athletes of high ability are on average more inclined to use drugs. At the individual level, however, socially dependent preferences allow for a refined view on doping, since there are some athletes of highest ability who refrain from doping if social disapproval is strong enough as well as some athletes of
lowest ability who succumb to doping if they get a lot of peer-group approval for a pro-doping decision.

The panel on the left hand side of Figure 1 shows the case where peer group approval of doping is so strong that there exists a critical level of susceptibility to approval above which all athletes dope irrespective of their ability. Formally, the curve represented by the threshold equation (3) hits the abscissa within the unit interval. This requires that \( S_t > S_{\text{high}} \equiv (c + \phi)/\beta \) such that \( \tilde{\sigma} = c/(\beta S_t - \phi) < 1 \) For given strength of approval \( S_t \) this case occurs when monetary costs \( c \) and stigma costs \( \phi \) are sufficiently low and cohesion of the athlete’s community \( \beta \) is sufficiently high.

The panel on the right hand side of Figure 1 shows the diametrically opposing case where peer approval is so low that there exists a critical level of susceptibility above which all athletes stay clean irrespective of their ability. The curve represented by threshold equation (3) has positive slope and assumes the value of unity within the unit interval. This case occurs when monetary and stigma costs are sufficiently high and peer group cohesion and the power of drugs \( \alpha \) are sufficiently low, i.e. for \( S_t < S_{\text{low}} \equiv (c + \phi - \alpha)/\beta \) such that \( \tilde{\sigma} = (\alpha - c)/(\phi - \beta S_t) < 1 \).

The central panel in Figure 1 shows the intermediate case for which at all levels of susceptibility to approval there are some doping athletes (of on average high ability) and some clean athletes (of on average low ability).

The area above the threshold in Figure 1 provides the share of doping athletes, denoted by \( \theta \). For the special case of socially-independent preferences, the threshold is given by the horizontal dashed line and the size of the area can be immediately read off the figure.

**Lemma 1.** For socially-independent preferences \( (\sigma(i) = 0 \text{ for all } i) \) the share of doping athletes is \( \theta_u = 1 - c/\alpha \).

In the case of socially-dependent preferences the share of doping athletes is situation-specific and depends on the strength of current peer-group approval for a pro-doping decision. Integrating the area above the threshold, we obtain the following piece-wise defined result.

\[
\theta_t = \theta(S_t) = \begin{cases} 
1 - \frac{c^2}{2\alpha(\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \equiv (\phi + c)/\beta \\
1 - \frac{1}{\alpha} \left[ c - \frac{1}{2}(\beta S_t - \phi) \right] & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
\frac{(\alpha - c)^2}{2\alpha(\phi - \beta S_t)} & \text{for } S_t \leq S_{\text{low}} \equiv (\phi + c - \alpha)/\beta.
\end{cases}
\]

(4)

The doping–approval association is in detail derived in the Appendix, which also contains a proof of the following Lemma about the slope of \( \theta(S_t) \).
Lemma 2. The share of doping athletes is everywhere increasing in the strength of peer-group approval, \( \theta'(S_t) > 0 \). The doping-approval association is convex for \( S_t < S_{low} \), concave for \( S_t > S_{high} \), and linear for intermediate \( S_t \).

Inspection of (4) provides the following result.

Proposition 1. For given strength of peer group approval \( S_t \) the incidence of doping depends positively on the power of drugs and of peer group cohesion (\( \partial \theta / \partial \alpha > 0, \partial \theta / \partial \beta > 0 \)). It depends negatively on the monetary and stigma costs of doping (\( \partial \theta / \partial c < 0, \partial \theta / \partial \phi < 0 \)).

So far, the observation of \( \theta(S_t) \) provides just a snapshot of the current incidence of doping in a sport. To find out in which doping situation a sport will end up under the prevalent rules and anti-doping policies we have to investigate existence, uniqueness, and stability of long-run equilibria.

3. Doping Cultures

On the aggregate level per group approval evolves endogenously. It depends positively on the share of athletes who were actually doping in the (recent) history of a sport. We assume that drug use is either directly observable at the team level or that athletes exchange their knowledge about doping. Let \( \delta \) denote the rate of oblivion by which the doping history of the sport is depreciated in the backward looking mind of athletes so that current approval is given by \( S_t = (1 - \delta) \sum_{i=0}^{\infty} \delta^i \theta_{t-1-i} \).

Alternatively, this can be written as the period-by-period evolution of approval,

\[
S_t = (1 - \delta) \cdot \theta_{t-1} + \delta \cdot S_{t-1}.
\]  

(5)

A social equilibrium is obtained where approval equals the actual incidence of doping, \( S_t = \theta_t \), such that the share of doping athletes stays constant over time. Inserting the equilibrium condition into (4) and solving for \( \theta \) provides the following result (proven in the Appendix).

Proposition 2. Depending on the size of parameters an athletes’ community is characterized by the following long-run incidence of doping. Let \( a \equiv (\alpha - c)\sqrt{2\beta/\alpha} \) and \( b \equiv \beta - c\sqrt{2\beta/\alpha} \).

I. If \( \phi > a \) and \( \phi > b \), then there exists a unique, globally stable equilibrium at \( \theta = \theta_{low} < \theta_u \),

\[
\theta_{low} \equiv \phi/(2\beta) - \sqrt{\phi^2/(4\beta^2) - (\alpha - c)^2/(2\alpha \beta)}.
\]

II. If \( a > \phi > b \), then there exists a unique, globally stable equilibrium at \( \theta = \theta_{mid} \), \( \theta_{mid} \equiv (\alpha - c - \phi/2)/(\alpha - \beta/2) \).

III. If \( b > \phi > a \), then there exist two locally stable equilibria \( \theta_{low} \) and \( \theta_{high} \) separated by an unstable equilibrium.
IV. If $\phi < a$ and $\phi < b$, then there exists a unique, globally stable equilibrium at $\theta = \theta_{\text{high}} > 1/2$, 

\[
\theta_{\text{high}} \equiv (\beta + \phi)/(2\beta) + \sqrt{(\beta - \phi)^2/(4\beta^2) - c^2/(2\alpha\beta)}.
\]

Figure 2: The Evolution of Doping Cultures

The result can best be explained with help of Figure 2. The four panels of the figure display the four qualitatively different curvatures and positions with respect to the identity line that the $\theta(S_t)$ curve of equation (4) can possibly assume. According to Lemma 2 a common feature of all four panels is that the $\theta(S_t)$-curve starts out with convex shape, becomes linear when $S$ exceeds $S_{\text{low}}$, and becomes concave when $S$ exceeds $S_{\text{high}}$. Note from inspection of (4) and Assumption 1 that $\theta(0) > 0$ and $\theta(1) < 1$. To alleviate comparisons, all four panels have been constructed by holding the individual costs and stigma costs of doping constant ($c = 0.4$ and $\phi = 0.3$ for all panels). Note also that social dynamics (5) lead the athletes’ community towards higher incidence of doping whenever the $\theta(S_t)$-curve lies above the identity line and towards lower incidence of doping when it lies below. The resulting dynamics are indicated by arrows on the $\theta_t$ axis.
Panel I visualizes case I of Proposition 2. In this case the power of performance enhancing drugs $\alpha$ and group cohesion $\beta$ are relatively low compared to individual and social costs ($\alpha = 0.5$ and $\beta = 1$). As a consequence, $\phi > a$ and $\phi > b$ and there exists just one equilibrium $\theta_{\text{low}}$. It turns out that socially-dependent preferences are actually helpful in reducing the incidence of doping. The social equilibrium lies below the one that would result if preferences were socially independent, $\theta_{\text{low}} < \theta_{\text{u}}$. The incidence of doping under socially-independent preferences is indicated by a star on the $\theta$ axis.

If, for some reason, the community started out at high $\theta$, peer-group approval and group cohesion are not strong enough to support such a high incidence of doping and some athletes are motivated to stay clean next period. A bandwagon dynamic (Granovetter, 1978) towards low $\theta$ sets in. At some point approval from peers $\beta S_t$ falls below social stigma $\phi$, and a pro-doping decision receives social disapproval. As a consequence the incidence of doping approaches a low value below $\theta_{\text{u}}$. Social disapproval, however, is not enough to eradicate doping entirely. Intuitively, there is always at least one athlete at the lower boundary of $\sigma$ who is not influenced by social disapproval and keeps using drugs as long as individual benefits are larger than individual costs (as long as $\alpha > c$). In conclusion, at $\theta_{\text{low}}$ performance enhancing drugs are mainly taken by a few independent-minded athletes who give not much on the social approval of their behavior.

Next consider Panel II, which differs from the setup of Panel I only by assuming a higher impact of doping on individual rank ($\alpha$ rises from 0.5 to 0.6). Given the higher power of drugs, peer-group approval is now sufficiently large to exceed stigma costs and the community of drug using athletes is able to generate support for an equilibrium $\theta_{\text{mid}}$. Comparing the intersection with the solution for socially-independent preferences (indicated by a star) shows that peer-group approval aggravates the doping problem. Recalling the results displayed in Figure 1 we can also infer that doping is more prevalent among athletes of high ability while those of low ability are more inclined to stay clean. Since net approval is positive at $\theta_{\text{mid}}$, the situation with respect to susceptibility to approval has reversed compared to Panel I. At $\theta_{\text{mid}}$ athletes who are easily influenced by peers are on average more inclined to use drugs.

Panel III differs from Panel I only by assuming a higher strength of group cohesion ($\beta$ rises from 1 to 1.5). While the equilibrium $\theta_{\text{low}}$ continues to exist, another locally stable equilibrium of high incidence of doping, $\theta_{\text{high}}$, emerges. It seems to be appropriate to speak of $\theta_{\text{high}}$ as a doping culture because, firstly, a majority of athletes uses performance enhancing drugs, $\theta_{\text{high}} > 1/2$. Secondly, most of the doping athletes are using drugs “only” because their competitors are using drugs as well. In order to see this, note that the power of drugs $\alpha$ is the same in Panel I and Panel III. It is thus
the high peer-group approval \( \beta S_t \) generated at \( \theta_{\text{high}} \) that makes the situation sustainable.

Panel III reflects the dilemma situation frequently referred to by professional athletes. At \( \theta_{\text{high}} \) the improvement in rank through doping is relatively small because a majority of competitors enhances their performance with drugs as well. As a result the majority of drug users would actually prefer to stay clean if only their competitors would refrain from doping as well. Formally, the same set of individual costs and benefits supports also an equilibrium \( \theta_{\text{low}} \). In order to reach this situation, however, a massive collective action is needed: a share \((\theta_{\text{high}} - \theta_{\text{mid}})\) of athletes has to coordinate to stay clean next season. Once the share of drug using athletes has fallen below \( \theta_{\text{mid}} \) there is enough social disapproval of a pro-doping decision to support a movement towards \( \theta_{\text{low}} \).

Finally, Panel IV differs from Panel I by both higher performance enhancing effect of drugs and tighter group cohesion \((\alpha \text{ rises to 0.6 and } \beta \text{ rises to 1.5})\), i.e. it differs from Panel III only by the higher power of drugs. Panel IV probably reflects best the situation in professional cycling, at least before the Festina scandal 1998. The power of drugs is strong enough to dispose of the equilibrium of low incidence of doping. A majority of athletes is doping and – with contrast to the situation in Panel III – collective action alone cannot manage to establish an equilibrium \( \theta_{\text{low}} \). In order to get rid of the doping culture athletes need assistance from sport organizations and/or the legislative.

Turning towards policy, taking the respective derivatives of \( \theta(S_t) \) proves the following result.

**Proposition 3.** Everywhere, i.e. for all \( S_t \), the share of doping athletes is decreasing in individual costs \((\partial \theta_t/\partial c < 0)\) and stigma costs \((\partial \theta_t/\partial \phi < 0)\). It is increasing in the power of drugs \((\partial \theta_t/\partial \alpha > 0)\) and in the strength of group cohesion \((\partial \theta_t/\partial \beta > 0)\).

Since the result applies everywhere, it holds also at the steady-state(s). These marginal comparative static effects are immediately intuitive. More interesting, however, are non-marginal effects of parameter changes:

**Proposition 4.** Any doping culture \( \theta_{\text{high}} \) can be eliminated by a sufficiently large increase of individual costs \( c \) or stigma costs \( \phi \) or by a sufficiently strong reduction of the power of drugs \( \alpha \) or the strength of group cohesion \( \beta \).

In order to be effective not only at the marginal level an anti-doping policy has to be sufficiently drastic. This, and other interesting aspects of policy, are visualized in Figure 3. Each of the three panels originates from the same initial situation, represented by the solid line and implying a long-run equilibrium at \( \theta_{\text{high}} \). Holding the power of drugs \( \alpha \) constant, the panels consider the effects of
increasing individual costs $c$ and stigma costs $\phi$ and of reducing group cohesion $\beta$.

The panel on the left hand side investigates rising individual costs of doping originating, for example, from higher fines or longer bans from competition when being exposed as a doper. If $c$ rises from 0.4 to 0.5 (dashed line), the athletes’ community moves towards a mildly lower equilibrium $\theta_{high}$ but the high incidence of doping in the sport does not disappear. In contrast to the initial situation, the rules under the higher costs of doping would, in principle, also support an equilibrium of low incidence of doping. But coming from a doping culture, an equilibrium $\theta_{high}$ remains to be (locally) supported by peer-group approval. In order to leave $\theta_{high}$ a more drastic increase of costs is needed. The dotted line shows the $\theta(S_t)$-curve for $c \to 0.6$. Since individual benefits and costs are now equalized, the only sustainable long-run situation is at $\theta_{low} \to 0$. Doping is eliminated.

Figure 3: Anti-Doping Policies

While increasing individual costs would be the only anti-doping policy available if preferences were socially-independent, social interaction allows to investigate two others policies. The central panel of Figure 3 shows that stigma costs of doping have effects similar to those obtained for individual costs. Again, the increase of costs must be sufficiently large in order to eliminate the doping culture $\theta_{high}$. The most salient difference compared with the $(c)$–Panel is that increasing stigma is less effective in manipulating $\theta_{low}$, i.e. when the incidence of doping is already low. Intuitively, at $\theta_{low}$ only the independent-minded athletes keep on using drugs. They are resistant to social stigma and need higher individual costs to quit doping.

12 Actually, a sports organization can also manipulate the power drugs, at least to a certain degree, by setting upper limits of acceptable substances detected in blood or urine tests.
A similar and even more pronounced outcome of an asymmetric effect of policy at $\theta_{\text{high}}$ and $\theta_{\text{low}}$ is visible in the right hand side panel, which investigates peer-group cohesion; $\beta$ is reduced from 1.5 to 1 (dashed lines) and to 0.6 (dotted lines). Conceivable policies bringing about such a change are leniency policies motivating doping convicts to testify against their teammates or a change of rules which reduces the dependence on teammates in competition (e.g. abandoning team time-trials in cycling). While a reduction of $\beta$ is very effective in eliminating a doping culture it is also completely ineffective in changing doping behavior at $\theta_{\text{low}}$. Intuitively, when the incidence of doping and thus peer-group approval is relatively low anyway ($S_t$ is low) it does not matter much how strongly athletes evaluate approval ($\beta S_t$ is low anyway).

The broad conclusion from these exercises is that the model supports the endeavor of the WADA and other sport organizations targeted on influencing the social environment of a sport. Measures trying to reduce group-cohesion and to raise the awareness of stigmatization of doping by society at large can be successful in eliminating a doping culture if they are sufficiently strong. But these measures are insufficient to eliminate doping entirely. For that an increase of individual costs is inevitable.\(^{13}\)

4. Rank Loss Aversion

Although the previous modeling of symmetric effects of doping and not-doping on rank may be appropriate for many sports, there are rules and specific situations in sports that produce asymmetric effects. This is in particular the case when the best performers set cut-off thresholds for all other competitors. A strong variant of such a threshold-rule exists in cycling at the Tour de France and other stage races. At each racing day the stage winner sets a cut off value on arrival time for all competitors. Riders arriving $x$ minutes later than the winner are excluded from the competition at further stages, i.e. in our notation they are automatically assigned with rank zero irrespective of their original ability. In particular, sprint specialists who could compete for high rankings (the Green Jersey) at later stages on flat land are threatened by elimination during the mountain stages.

In many other sports the cut-off phenomenon occurs not as a general rule as in cycling but is situation-dependent. For example, at the national level the best athlete in a sport sets a threshold value for participating at the Olympic Games. In all these cases a drug using winner, an athlete of anyway high ability, executes a disproportionately high threat on the career of clean athletes even

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\(^{13}\)Acknowledging the power of social influence on the formation of athletes’ attitudes and beliefs the WADA has recently launched the “Play True Generation” program (WADA, 2008). Similar programs trying to increase an athletes awareness of the size of $\phi$ and to reduce $\beta$ are launched on the national level (e.g. the “100% Me” program in the UK) and at the level of sport organizations (e.g. the “True Champion or Cheat” program of the UCI).
if they are not aspiring to win but only to participate.

In order to capture asymmetric effects we introduce the parameter λ ≥ 1 into the rank function (1), which becomes \( R_t(i, d) = A(i) \cdot \{1 + \alpha [d \cdot (1 - \theta_t) - \lambda \cdot (1 - d) \cdot \theta_t]\} \). The larger \( \lambda \) the larger the rank loss if an athlete stays clean relative to the rank gain of a doping athlete. Inserting the modified rank function into the utility function provides

\[
U(i, d) = A(i) \cdot \{1 + \alpha [d \cdot (1 - \theta_t) - \lambda \cdot (1 - d) \cdot \theta_t]\} - d \cdot c + d \cdot \sigma(i) \cdot (\beta S_t - \phi).
\]

Proceeding as in Section 2 we compare for any athlete \( i \) utility when doping and not doping and obtain the modified ability–susceptibility threshold between clean and doping athletes.

\[
A = \frac{c - \sigma(\beta S_t - \phi)}{\alpha [1 + \theta(\lambda - 1)]}. \tag{7}
\]

Again, athletes characterized by an ability–susceptibility tuple \((A, \sigma)\) above the threshold prefer doping and athletes with an \((A, \sigma)\) tuple below the threshold stay clean. Inspecting (7) we see that the right hand side decreases as \( \lambda \) rises. This implies that a season \( t \) snapshot confirms the “weak athlete argument”: the higher the threat of rank loss \( \lambda \) and the higher the share of doping athletes \( \theta_t \), the lower the threshold that has to be crossed in order to enter the club of doping athletes.

In contrast to the basic model we can no longer explicitly solve for the equilibrium share of doping athletes \( \theta_t \). Instead we arrive at an implicit function determining \( \theta_t \) for given \( S_t \).

\[
0 = G(\theta_t, S_t) = \begin{cases} 
1 - \theta_t - \frac{c^2}{2\alpha[1+\theta_t(\lambda-1)](\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \equiv (\phi + c)/\beta \\
1 - \theta_t - \frac{1}{\alpha[1+\theta_t(\lambda-1)]} [c - \frac{1}{2}(\beta S_t - \phi)] & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \tag{8} \\
\frac{\alpha[1+\theta_t(\lambda-1)]-c}{2\alpha[1+\theta_t(\lambda-1)](\phi-\beta S_t)} - \theta_t & \text{for } S_t \leq S_{\text{low}} \equiv \frac{\phi+c-\alpha[1+\theta_t(\lambda-1)]}{\beta}.
\end{cases}
\]

The Appendix derives (8) and proves the following result.

**Proposition 5.** At any long-run equilibrium \( \theta^* \) the incidence of doping is increasing in the relative size of rank loss when not doping \( (\partial \theta^*/\partial \lambda > 0) \). The incidence of doping is increasing in the power of drugs \( \alpha \) and group cohesion \( \beta \) and decreasing in individual and social costs \( (c \text{ and } \phi) \).

Rank loss aversion does not only aggravate the doping problem. It may actually be responsible for doping to exist. The easiest way to see this, is to consider the case of socially-independent preferences \((\sigma = 0 \text{ for all athletes})\). The threshold (7) then reduces to \( A = c/[1 + \theta_t(\lambda - 1)] \). Integrating the area above the threshold provides the quadratic equation \( \theta_t^2(\lambda - 1) - \theta_t(\lambda - 2) - 1 + c/\alpha = 0 \). Given
the constraint that $\theta \in [0,1]$, this provides a unique solution $\theta_u$.\textsuperscript{14} Now consider $c \to \alpha$. In the basic model, without rank loss aversion, such a rise of costs eliminates doping entirely. With rank loss aversion, however, the solution is $\theta_{u,\lambda} = \max \{0, (\lambda - 2)/(\lambda - 1)\}$, which is strictly positive for $\lambda > 2$. In conclusion, rank loss aversion is sufficient to explain the incidence of doping,

**Figure 4: The Impact of Rank Loss Aversion on Doping and Policy Effectiveness**

The fact that preferences are socially-dependent amplifies the problem of rank loss aversion. It raises the incentive to dope everywhere and may even eliminate the possibility of an equilibrium of low incidence of doping. These conclusions are illustrated in Figure 4. The $(\lambda)$-panel on the left hand side resumes case III of Figure 2 and displays the effect of increasing rank loss aversion. The solid line reiterates the case without rank loss aversion ($\lambda = 1$), characterized by two locally stable equilibria $\theta_{\text{high}}$ and $\theta_{\text{low}}$. The dashed line shows the consequence of $\lambda = 2$. The emergence of rank loss aversion, e.g. through the introduction of minimum arrival times at stage trials, eliminates the equilibrium $\theta_{\text{low}}$. The fear of rank loss is sufficient to move an athlete’s society from an equilibrium of low incidence of doping towards a doping culture $\theta_{\text{high}}$.

Intuitively, starting at $\theta_{\text{low}}$ there are initially only a few athletes who take up doping after introduction of the new, rank loss aversion generating rules. Diagrammatically at low peer-group approval $S_t$ the dotted lines is just above the 45-degree line, such that $\theta_{\text{low}}$ ceases to exist. Next a bandwagon effect sets in. Since more athletes are using drugs, peer group approval rises, and further athletes are motivated to take drugs. Additionally, some athletes start using drugs to avoid or mitig-

\textsuperscript{14} The introduction of asymmetries in the utility function is thus not sufficient to generate multiple equilibria. Socially-dependent preferences remain to be essential for multiple equilibria to occur.
gate losing rank because of their increasingly doped competitors. Their behavior in turn amplifies peer-group approval further, etc. As consequence the athletes’ community ends up at an equilibrium \( \theta_{\text{high}} \) where the incidence of doping is higher than it would be without rank loss aversion. Further rising rank loss aversion exacerbates the problem but does not qualitatively change the picture. For \( \lambda \to \infty \) the \( \theta(S_t) \) curve converges towards a step function assuming the value of unity for all \( S_t \).

The panel on the right hand side of Figure 4 resumes the policy experiment from Figure 3. The solid line reflects the same parameters as the solid line in Figure 3 except that \( \lambda = 2 \). Comparing the \((c)\) panels of Figure 3 and 4 leads to the conclusion that rank loss aversion drastically reduces the scope of anti-doping policy. An increase of individual costs \( c = 0.5 \) (dashed lines) does not change the situation under rank loss aversion. Without rank loss aversion it has led to the emergence of \( \theta_{\text{low}} \) and the hope that a collective action effort moves the athletes’ community towards a low-doping equilibrium. This possibility arises under rank loss aversion only if costs are further increased toward \( c \to 0.6 \), a policy that has eliminated doping entirely in the basic model.

5. The Taste for Victory

It has been frequently argued that the unequal distribution of honors (and money) is at the root of all evil in professional sports. For example, at the Olympic games there is just one gold medal and only three medals altogether for any discipline. Already the fourth in competition returns home with empty hands, more or less just as every other participant. If the Olympic motto “participation is everything” was ever a social norm it is long gone and perhaps replaced by “winning is everything”.

We discuss disproportionate effects from the aspiration to stay on top by modifying the utility function so that agent \( i \) gets utility \( R(i, d)^\gamma \) out of his rank, \( \gamma \geq 1 \). The higher \( \gamma \) the higher the disproportionate effect of rank on utility. For \( \gamma \to \infty \) utility converges towards a step function, where the winner receives a utility value of one and all others athletes receive no utility at all, i.e. preferences converges towards the case where “winning this the only thing.”

Taking disproportionate utility from rank into account we rewrite (2) as

\[
U(i, d) = \left( A(i) \cdot \left\{1 + \alpha [d \cdot (1 - \theta_t) - (1 - d) \cdot \theta_t]\right\}\right)^\gamma - d \cdot c + d \cdot \beta \cdot \sigma(i) \cdot (S_t - \phi).
\]

Athletes compare the solutions of (9) for \( d = 0 \) and \( d = 1 \) in their decision whether to use drugs or stay clean. From that we see that athlete \( i \) refrains from doping if \( A(i)^\gamma z_t \leq c - \sigma(i) [\beta S_t - \phi] \) with \( z_t \equiv \{[1 + \alpha(1 - \theta_t)]^\gamma - [1 - \alpha \theta_t]^\gamma\}^{1/\gamma} \). The ability-susceptibility threshold dividing clean and
doping athletes is thus given by
\[ A = \frac{c - \sigma(\beta S_t - \phi)}{z_t} \]  
(10)

Again, we cannot solve explicitly for the incidence of doping but get it determined by an implicit function.

\[ 0 = F(\theta_t, S_t) = \begin{cases} 
1 - \theta_t - \frac{\gamma c^{1+1/\gamma} - (c+\phi-\beta S_t)^{1+1/\gamma}}{(1+\gamma)(\beta S_t-\phi)z_t} & \text{for } S_t \geq S_{\text{high}} \equiv (\phi + c)/\beta \\
1 - \theta_t - \frac{\gamma c^{1+1/\gamma} - (c+\phi-\beta S_t)^{1+1/\gamma}}{(1+\gamma)(\beta S_t-\phi)z_t} & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
1 - \theta_t - \frac{\gamma c^{1+1/\gamma} - (c+\phi-\beta S_t)^{1+1/\gamma}}{(1+\gamma)(\beta S_t-\phi)z_t} & \text{for } S_t \leq S_{\text{low}} \equiv \frac{\phi + c - z_t^\gamma}{\beta}. \end{cases} \]

(11)

Interestingly, a higher taste for victory does not necessarily lead to a higher share of drug-using athletes in equilibrium. Moreover, a sufficiently high increase of the taste for victory eliminates both an equilibrium of low incidence of doping \( \theta_{\text{low}} \) and a doping culture \( \theta_{\text{high}} \) and initiates a move towards a globally stable equilibrium \( \theta_{\text{mid}} \).

These results are shown in the panel on the left hand side of Figure 5. The solid line reiterates case III from Figure 2. Suppose, initially the athletes’ community is situated at or close to \( \theta_{\text{low}} \). The dashed line shows the effect of a rising taste for victory from \( \gamma = 1 \) to \( \gamma = 2 \). The higher veneration of winners eliminates the \( \theta_{\text{low}} \) equilibrium because a larger share of high-ability athletes takes up doping. The motivation of these athletes originates predominantly from the higher desirability of a high rank, i.e. it occurs irrespective of peer-approval. This can be seen in the diagram by the upward shift of the lower part of the \( \theta(S_t) \) curve.

The initiated bandwagon effect moves the athletes’ community towards a doping culture \( \theta_{\text{high}} \). The incidence of doping at \( \theta_{\text{high}} \) is somewhat lower than it would be without the higher taste for victory because some athletes of low ability are discouraged from taking drugs. They refrain from doping because they would anyway – with or without assistance of performance enhancing drugs – not be able to reach the highest ranks.

In fact, the taste for victory can discourage so many athletes of low ability from taking drugs that the doping culture \( \theta_{\text{high}} \) becomes unsustainable. This is shown by the dotted line representing the case of \( \gamma = 10 \), i.e. a very strong preference for attaining the highest ranks. Many athletes of lesser ability are abandoning drugs and the reduced peer-group approval caused by their abstention eliminates \( \theta_{\text{high}} \). However, instead of approaching \( \theta_{\text{low}} \) the athletes’ community arrives at an intermediate equilibrium \( \theta_{\text{mid}} \) where athletes of high ability continue to use drugs in order to compete for the highest ranks. Their decision is relatively independent from social approval, a fact that can be inferred from the relatively flat slope of the lower part of the \( \theta(S_t) \) curve.
The observation that at such a “superhero-equilibrium” athletes of highest ability are trying to further push their rank whereas the weak and intermediate athletes stay clean has an empirically verifiable implication. If we look at results in absolute terms (for example, arrival time at a mountain stage at the Tour de France) we should be able to observe a clear structural break between the best athletes in the field and the rest. Interestingly, in the year 2007 this was indeed observed at the Tour de France. The French press spoke of “le cyclisme a deux vitesses”, two-speed cycling: one subset of riders climbed the mountains at incredible speed and sometimes seemingly without visible effort, and the rest of the peloton was left behind.\footnote{A similar phenomenon was observed twice before. In 1999, after the Festina scandal, when the French cycling association unilaterally implemented stricter anti-doping rules, and in 2002, after reliable tests for EPO doping became available, which (allegedly) induced a few high ability riders (the team leaders) to turn towards the much more elaborate and expensive blood doping and caused a huge gap between the best riders and the rest of the peloton.}

The superhero equilibrium turns out to be rather resistant against policy. With respect to the parameters comprising social influence, $\beta$ and $\phi$ this conclusion is obvious from the flat slope of $S(\theta)$. The panel on the right hand side of Figure 5 demonstrates it is also true with respect to individual costs. The panel resumes the scenario analyzed by the (c)-panels of Figures 3 and 4, keeping the same parameters except that now $\gamma = 10$, indicating a very high taste for victory. The effect of an increase of individual costs is reflected by the dashed line($c$ rises from 0.4 to 0.5) and the dotted line ($c = 0.6$). One sees that even the formerly quite effective large increase of costs has now only little effect on the incidence of doping and no power to eliminate $\theta_{\text{mid}}$. While the majority of professional athletes continuous to react on increasing costs (more controls, heavier punishment) with abandoning doping, a minority of superheroes resists.
6. Ability-dependent Costs of Doping

There are certainly good reasons to assume that costs are lower for high-ability athletes, for example, because they have better contacts for drug acquisition or because drugs are sponsored by third parties (see footnote 15). A negative ability-cost correlation, however, would not change any results. In fact, it would make the conclusion even more compelling that doping is particular prevalent among the ablest athletes. It is thus more interesting to explore the consequences of a positive correlation between ability and costs, an assumption which has the potential to reverse or qualify the results obtained so far. A positive ability-cost correlation could be motivated by the notion that athletes of high ability have possibly more to lose in terms of expected income when identified as a doper.

In the following we exemplarily focus on quadratic costs because they allow for an explicit solution.\textsuperscript{16} Suppose the costs of doping for athlete \( i \) are \( c + \eta A(i)^2 \) with \( c > 0 \) and \( \eta > 0 \). It is illustrative to consider for a moment socially-independent preferences (\( \sigma(i) = 0 \) for all \( i \)). Then, athletes stay clean if \( \alpha A(i) < c + \eta A(i)^2 \). The interesting case here emerges when the quadratic equation has two real solutions, which is the case for sufficiently strong power of drugs, i.e. for \( \alpha^2 > 4c\eta \). In that case we get a split athletes’ community. Athletes stay clean if

\[
A(i) \leq \max(0, A_1) \quad \text{or} \quad A(i) \geq \min(1, A_2)
\]

with \( A_1 \equiv \alpha/(2\eta) - \sqrt{s} \) and \( A_2 \equiv \alpha/(2\eta) + \sqrt{s} \), \( s \equiv \alpha^2/(4\eta^2) - c/\eta \). This means that, in contrast to the model variants discussed so far, athletes of intermediate ability are most heavily inclined to use drugs. While low-ability athletes are not getting enough rank improvement from doping, high-ability athletes refrain from doping because of the entailed high costs. In the following we focus on the case of a split athletes’ community because it is the only case that constitutes a qualitative modification of the basic model. It implies the following restriction on parameters, which replaces Assumption 1.

**Assumption 2.** \( \eta + c > \alpha \geq 2\sqrt{cn} \).

The first part assumes that costs are sufficiently strongly increasing in ability such that there exists at least one clean athlete of highest ability, i.e. \( \eta > \alpha - c \), ensuring that \( A_2 < 1 \). The second part assumes that drugs are powerful enough for doping to exist if preferences were independent from social (dis-) approval of doping, i.e. \( \alpha^2 \geq 4c\eta \), ensuring real solutions \( A_1 \) and \( A_2 \). From existence of \( A_1 \) follows \( A_1 > 0 \) since \( c > 0 \) and \( \eta > 0 \).\textsuperscript{17}

\textsuperscript{16}A linear association between ability and costs would not change any of the conclusion obtained so far.

\textsuperscript{17} Note that Assumption 2 also implies that \( \eta + c > 2\sqrt{cn} \Leftrightarrow (\eta - c)^2 > 0 \) which is fulfilled for all \( c \neq \eta \).
Facing quadratic costs, athlete \( i \) stays clean if \( \alpha A(i) < c + \eta A(i)^2 - \sigma(i) [\beta S_t + \phi] \). Given the non-monotonous association between ability and drug use, it is helpful to swap axes of the ability-susceptibility space because the threshold can then be represented as a function \( \sigma(A) \), which can be easily integrated to obtain the incidence of doping. This way, the threshold is given by

\[
\sigma = \frac{A [\eta A - \alpha] + c}{\beta S_t - \phi}.
\]  

As shown in Figure 6 the threshold (12) has two roots given by the boundaries of the doping range under socially-independent preference. Inspecting the slope of the threshold, \( \partial \sigma / \partial A = (2 \eta A - \alpha) / (\beta S_t - \phi) \), identifies an extremum at \( A = \alpha / (2 \eta) \). If peer-group approval is sufficiently high, \( S_t > S_{low} \equiv \phi / \beta \), the extremum is a minimum, as in the left and central panel of Figure 6. It implies that for given ability the incentive to use drugs is higher among athletes of high-susceptibility to social approval. Similarly to the results from the basic model, we can further distinguish a case where subgroups of athletes refrain from doping completely. If \( S_t < S_{high}^1 \equiv (c + \phi) / \beta \) then there exists a range of athletes of lowest ability who are not doping, \( b_1 = \alpha / (2 \eta) - \sqrt{x} \), \( x \equiv \alpha^2 / (4 \eta^2) - (c + \phi - \beta S_t) / \eta \). On the other side of the ability spectrum, if \( S_t < S_{high}^2 \equiv (c + \phi + \eta - \alpha) / \beta \), there exists a range of athletes of highest ability who are not doping, \( 1 - b_2 \equiv 1 - \alpha / (2 \eta) - \sqrt{x} \). This case is shown in the central panel of Figure 6.

Finally, if \( S_t \) is even lower, \( S_t < S_{low} \), social approval turns into disapproval, the extremum becomes a maximum, and the threshold flips around in the \( A-\sigma \)-diagram. This case is shown in the panel on the right hand side of Figure 6. For given ability, athletes who are largely immune against social approval are now most heavily inclined to use drugs.

In order to obtain the share of doping athletes we integrate the area below the threshold, and
solve for $\theta$. This provides

$$\theta = \begin{cases} 
B_2 - B_1 - \frac{1}{\beta S_t - \phi} \left[ \frac{\eta}{3} \left( A^3_1 - B^3_1 + B^3_2 - A^3_2 \right) - \frac{\alpha}{2} \left( A^2_1 - B^2_1 + B^2_2 - A^2_2 \right) + c \left( A_1 - B_1 + B_2 - A_2 \right) \right] \\
\frac{1}{\beta S_t - \phi} \left[ \frac{\eta}{3} \left( A^3_2 - A^3_1 \right) - \frac{\alpha}{2} \left( A^2_2 - A^2_1 \right) + c \left( A_2 - A_1 \right) \right]
\end{cases}$$

for $S_t \geq S_{low} \equiv \phi/\beta$

$$\left[ \frac{\eta}{3} \left( A^3_3 - A^3_1 \right) - \frac{\alpha}{2} \left( A^2_3 - A^2_1 \right) + c \left( A_3 - A_1 \right) \right]$$

for $S_t \leq S_{low}$.

(13)

where $B_1 = \max \left\{ 0, \alpha/(2\eta) - \sqrt{x} \right\}$, $B_2 = \min \left\{ 1, \alpha/(2\eta) + \sqrt{x} \right\}$, $x \equiv \alpha^2/(4\eta^2) - (c + \phi - \beta S_t)/\eta$.

Figure 7 shows the effectiveness of anti-doping policy. The initial situation (solid lines) resumes the scenario of Figure 3 and assumes parameters supporting a doping culture $\theta_{high}$. The panel on the left hand side considers ability-independent costs of doping. Obviously the cost increase has exactly the same effects as in the basic model although now athletes of intermediate ability are most heavily inclined to use drugs. A medium-sized increase of $c$ (dashed line) generates multiple equilibria and a sufficiently drastic increase (dotted line) initiates an escape from the doping culture.

An increase of the new policy variable $\eta$ leads to very similar results, as shown in panel on the right side of Figure 7. A medium-size increase produces multiple equilibria and a drastic increase initiates convergence towards $\theta_{low}$. Both policies, however, differ greatly on the individual level. An increase of $c$ (for example, the monetary fine to be paid if being identified as a doper) affects athletes of the entire ability spectrum whereas an increase of $\eta$ (for example, the length of a ban from participation) discourages doping predominantly among athletes of high (but not highest) ability.

Intuitively, one would expect a particularly strong rise of $\eta$ through the introduction of retroactive testing and punishment because it opens the possibility to get ex post deprived of victories and prize
money. Yet the first cases of retroactive tests in cycling have actually exposed high-ability riders as dopers (2008 Tour stage winners Leonardo Piepoli and Stefan Schumacher and overall third Bernard Kohl using a then undetectable EPO-derivate). This indicates that the performance enhancing power of EPO and blood doping are just too strong and that the basic model and, perhaps, the taste-of-victory variant are more appropriate to describe the doping situation in cycling.

7. Conclusion

The proposed theory has shown how community dynamics can move a sport towards an equilibrium where a large majority is using drugs without getting much out of it ranking-wise. Not because of sudden moral qualms but because costly doping has become so inefficient a situation is reached where it is in the self-interest of the majority of professional athletes to get rid of the doping culture.

In the case of multiple equilibria, athletes could, in principle, release themselves from the equilibrium of high incidence of doping. This would require a strong collective action, i.e. a one-time drastic reduction of $\theta_t$, so that the system leaves the domain of attraction of the equilibrium of high incidence of doping. Generating such a collective action, however, may be too hard to be managed by athletes without external help. In this case, and in any case if the equilibrium of high doping incidence is unique, a change of the rules of the game is needed in order to initiate a movement towards low or absent doping.\footnote{For example, a collective action was tried by eight French and German teams at the Tour de France 2007. They formed the so called “Movement for Credible Cycling”, subjected themselves to additional, voluntary anti-doping surveillance, and organized sit-down strikes against doping before the beginning of stages. Since, nevertheless, some riders of the “Movement” were caught doping, the idea was not successful.}

It has been shown that an increase of individual costs eliminates a doping culture only if it is sufficiently drastic. Besides this perhaps obvious conclusion, it has been shown that similar effects can be expected from a sufficiently strong increase of stigma costs and from a reduction of group cohesion. The theory thus supports recently launched educational programs designed to reduced peer-group influence (e.g. WADA, 2009a, UCI, 2009). But it also predicts that – with contrast to increasing individual costs – one cannot expect from these programs to eliminate doping entirely.

An extension of the model has shown why rank loss aversion generated, for example, by qualification marks set be the best – and presumably doped – athletes in a competition increases the prevalence of doping and may actually eliminate an equilibrium of low incidence of doping. It has also been shown that such rules reduce the power of anti-doping policies. In order to eliminate a doping culture the theory thus recommends to abandon or to reduce standards for participation that are set by the best athletes in a sport. Certainly, this is more easily achieved in some sports (stage...
disqualification cut offs in cycling) than in others (qualification marks for the Olympic games).

A further modification of the model has shown that a disproportionate emphasis of winning (or, more generally, finishing among the top athletes) greatly reduces the power of anti-doping policies because a subgroup of most talented competitors in a sport is largely resistant against rising monetary costs and social costs of doping. If a sport is situated in such a superhero-equilibrium the anti-doping strategy of highest priority should be to downplay the role of winners and top finishers by, for example, distributing money and tv appearances more equally among all ranks.

Finally, it could be useful to think of cycling not as a special, particularly rotten sport but as a precursor of the things to come in other disciplines. Because doping has been so effective in increasing performance and because of the long history of doping in the sport and the thereby generated peer-group approval, the social dynamics may have carried the cyclists’ community close towards an equilibrium of particularly high incidence of doping. In the future, when gene doping allows for pronounced improvements of performance in every kind of physical activity, other sports will have the advantage to learn from the development of doping and anti-doing policies in cycling.
Appendix

Derivation of (4). The critical $\sigma$ above which all athletes dope is obtained by setting $A = 0$ in (3), which provides $\sigma = c/ (\beta S_t - \phi) \equiv \sigma$. For $\sigma$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \geq (c + \phi)/\beta \equiv S_{\text{high}}$. Likewise, when $S_t < \phi$, the critical level of $\sigma$ above which all athletes stay clean is obtained by setting $A = 1$ in (3), which provides $\sigma = (\alpha - c)/(\phi - \beta S_t) \equiv \sigma$. For $\bar{\sigma}$ to exist within $[0, 1]$ it has to be smaller or equal to unity, requiring that $S_t \leq (c + \phi - \alpha)/\beta$.

Generally, integrating the area below the threshold (3) within the limit $0$ and $x$ provides

$$\int_0^x \frac{c - \sigma (\beta S_t - \phi)}{\alpha} d\sigma = \frac{1}{\alpha} \left[ c \sigma - \frac{\sigma^2}{2} (\beta S_t - \phi) \right]_0^x.$$

If $S_t > S_{\text{high}}$, integrate up to $x = \bar{\sigma}$ to obtain

$$1 - \theta = \frac{c^2}{2 \alpha (\beta S_t - \phi)}.$$

Likewise for $S_{\text{low}} < S_t < s_{\text{high}}$, integrate up to $x = 1$ to obtain

$$1 - \theta = \frac{1}{\alpha} \left[ c - \frac{1}{2} (\beta S_t - \phi) \right].$$

The case $S_t \leq S_{\text{low}}$ is a little more involved. To obtain $1 - \theta$ integrate up to $x = \bar{\sigma}$ and add $(1 - \bar{\sigma})$, i.e. the share of athletes who stay clean irrespective of $\sigma$. This provides

$$1 - \theta = \frac{1}{\alpha} \left[ \frac{c (\alpha - c)}{(\phi - \beta S_t)} + \frac{(\alpha - c)^2}{2 (\phi - \beta S_t)} \right] + 1 - \frac{\alpha - c}{(\phi - \beta S_t)} = 1 - \frac{(\alpha - c)^2}{2 \alpha (\phi - \beta S_t)}.$$

Collecting terms, we get (4).

Curve Discussion of (4). At $S_t = 0$ we have $\theta(0) = (\alpha - c)^2/(2 \alpha \phi) > 0$. At $S_t = 1$ we have $\theta(1) = 1 - c^2/(2 \alpha (\beta - \phi)) < 1$. Observe that $\theta(S_t)$ is continuous in $[0, 1]$. The first derivative is

$$\frac{\partial \theta(S_t)}{\partial S_t} = \begin{cases} \frac{c^2 \beta}{2 \alpha (\beta S_t - \phi)^2} & \text{for } S_t \geq S_{\text{high}} \\ \frac{\beta}{2 \alpha} & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\ \frac{(\alpha - c)^2 \beta}{2 \alpha (\phi - \beta S_t)^2} & \text{for } S_t \leq S_{\text{low}}. \end{cases}$$

and thus everywhere positive. The second derivative is

$$\frac{\partial^2 \theta(S_t)}{\partial S_t^2} = \begin{cases} -\frac{c^2 \beta^2}{2 \alpha (\beta S_t - \phi)^2} < 0 & \text{for } S_t \geq S_{\text{high}} \\ 0 & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \end{cases}$$
and
\[
\frac{\partial^2 \theta(S_t)}{\partial S_t^2} = \frac{(\alpha - c)^2 \beta^2}{2\alpha(\phi - \beta S_t)^3} > 0 \quad \text{for } S_t \leq S_{\text{low}}.
\]
The curve is thus convex at its lower part, linear at its middle part, and concave at its upper part.

**Proof of Proposition 2.** To begin with consider possible equilibria along the convex part of the \(\theta(S_t)-\)curve, Setting \(S_t = \theta\) in (4) when \(S_t < S_{\text{low}}\) provides \(\theta = (\alpha - c)^2/(2\alpha(\phi - \beta S_t))\). Solving for \(\theta\) we get two solution candidates.

\[
\theta_{1,2} = \frac{\phi}{2\beta} \pm \sqrt{\frac{\phi^2}{4\beta^2} - \frac{(\alpha - c)^2}{2\alpha\beta}} \quad (A.1)
\]

Existence of a solution requires that the radicand is non-negative, i.e. \(\phi > a \equiv (\alpha - c)\sqrt{2\beta/\alpha}\). For being assumed as a solution along the convex part the equilibrium \(\theta = S\) has to be smaller than the upper boundary of the convex segment \(\theta(S_{\text{low}})\). Inserting \(S_{\text{low}}\) into the convex segment of (4) we obtain the upper boundary \(\theta(S_{\text{low}}) = (\alpha - c)/(2\alpha)\). For the smaller root to be feasible this implies that

\[
\frac{\phi}{2\beta} - \sqrt{\frac{\phi^2}{4\beta^2} - \frac{(\alpha - c)^2}{2\alpha\beta}} < \frac{\alpha - c}{2\alpha} \quad \Rightarrow \quad (\alpha - c)\left(1 + \frac{\beta}{2\alpha}\right) < \phi.
\]
Likewise for the larger root to be smaller than \(\theta(S_{\text{low}})\) we obtain the condition that \((\alpha-c)(1 + \beta/(2\alpha)) > \phi\). Because the two conditions are mutually exclusive, there exists at most one equilibrium \(\theta_{\text{low}}\). Since the \(\theta(S_t)\) curve starts out above the 45 degree line, it cuts the 45 degree line first at the smaller root. If this equilibrium exists, then there exists no other equilibrium along the convex segment. This provides \(\theta_{\text{low}}\) of Proposition 2.

To see that \(\theta_{\text{low}} < \theta_u\) is to notice that the maximum that \(\theta_{\text{low}}\) can potentially assume is at the boundary of the convex segment, i.e. at \(\theta(S_{\text{low}})\) and that

\[
\theta(S_{\text{low}}) = \frac{\alpha - c}{2\alpha} < \frac{\alpha - c}{\alpha} \equiv \theta_u.
\]

Next consider equilibria along the intermediate, linear part of the \(\theta(S_t)\) curve. Because of linearity, there exists at most one equilibrium. Setting \(S_t = \theta\) in (4) when \(S_{\text{low}} < S_t < S_{\text{high}}\) and solving for \(\theta\) provides

\[
\theta_{\text{mid}} = \frac{\alpha - c - \phi/2}{\alpha - \beta/2} \quad (A.2)
\]

Finally consider equilibria along the upper, concave part of the \(\theta(S_t)\) curve. Setting \(S_t = \theta\) in (4) when \(S_t > S_{\text{high}}\) and solving for \(\theta\) provides solution candidates

\[
\theta_{3,4} = \frac{\beta + \phi}{2\beta} \pm \sqrt{\frac{(\beta - \phi)^2}{4\beta^2} - \frac{c^2}{2\alpha\beta}} \quad (A.3)
\]
Again, existence of a solution requires a non-negative radicand, which requires that \( \phi < b \equiv \beta - c\sqrt{2\beta/\alpha} \). Let \( \theta_{\text{high}} \) denote the larger root.

In order to verify that \( \theta_{\text{high}} > 1/2 \) begin with noting that \( \theta_{\text{high}} \) is assumed along the concave segment of which the lower boundary is given by \( S_{\text{high}} \). Thus \( \theta_{\text{high}} \geq \theta(S_{\text{high}}) \). Insert \( S_{\text{high}} \) into (4) to obtain \( \theta(S_{\text{high}}) = 1 - c/(2\alpha) \) which exceeds 1/2 because \( 1 > c/\alpha \).

We have now gathered all elements needed for the discussion of existence, uniqueness, and stability of equilibria of the \( \theta(S_t) \) curve. Figure A shows all possible positions of the convex, linear, and concave segments of the curve with respect to the 45 degree line. The 45 degree line, along which \( \theta_t = S_t \), identifies equilibria. Note from (4) that \( \theta(0) > 0 \) and \( \theta(1) < 1 \). Along the concave part there exist thus either one (1.b) or two (1.c) equilibria, or the convex part lies entirely below the 45 degree line (1.a). The linear part lies either entirely above (2.a) or below (2.b) the 45 degree line, or there exists a unique intersection, either from above (2.c) or from below (2.d). The convex part originates from positive \( \theta(0) \) and intersects the 45 degree line either once (3.b) or not at all (3.a).

There are “only” four qualitatively different permutations of segments. To see this, begin with noting from (4) that the \( \theta(S_t) \) curve is everywhere continuous in the \((0, 1)\) interval implying smooth pasting of the individual segments.

Figure A: Elements of the \( \theta(S_t) - Curve \)

To begin with, consider segment 1.a, which can be pasted smoothly to segments 2.b or 2.c.
Consider first the sequence 1.a–2.b. It can be pasted smoothly only to 3.b. The sequence 1.a.-2.b.-3.b establishes Case I of Proposition 2 visualized in Panel I of Figure 2. As shown above, for Case I to hold we must have that \( \phi > a \) such that there exists an equilibrium along the lower convex part, and \( \phi > b \) such that there exists no equilibrium along the concave part.

The sequence 1.a-2.c can be pasted smoothly only with segment 3.a. The emerging sequence 1.a.-2.c-3.a establishes Case II of Proposition 2 visualized in Panel II of Figure 2. This case identifies a unique equilibrium \( \theta_{\text{mid}} \) assumed along the linear part of the curve. For this case to occur there must be no equilibrium along the upper concave part, which – as shown above – requires \( \phi > b \) and no equilibrium along the lower convex part of the curve, which requires \( \phi < a \).

Next consider segment 1.b. It can be pasted smoothly with segments 2.a and 2.d. Consider first the sequence 1.b.-2.a. It can be pasted smoothly only with 3.a. The sequence 1.b-2.a-3.a constitutes Case IV of Proposition 2 visualized in Panel IV of Figure 2. It identifies a unique equilibrium \( \theta_{\text{high}} \) assumed along the upper, concave segment of the \( \theta(S_t) \) curve. For this case to hold we must have \( b > \phi \), such that there exists an equilibrium along the concave part of the curve, and \( a > \phi \) such that there exists no equilibrium along the convex part of the curve.

The sequence 1.b.-2.d. can be pasted smoothly only with segment 3.b. The emerging sequence 1.b.-2.d-3.b establishes Case III of Proposition 2 visualized in Panel III of Figure 2. Here we have three equilibria, which requires \( \phi > a \) for \( \theta_{\text{low}} \) to exist and \( \phi < b \) for \( \theta_{\text{high}} \) to exist.

Finally consider segment 1.c, i.e. the case where there exist two equilibria along the concave segment of the curve. It can be match to the sequence 2.c-3.a or to the sequence 2.b-3.b. These sequences identify variants of Case III, where we have three equilibria. As already shown above, we must have \( \phi > a \) for \( \theta_{\text{low}} \) to exist and \( \phi < b \) for \( \theta_{\text{high}} \) to exist.

Note that the social dynamics (5) lead to increasing \( \theta \) when the \( \theta(S_t) \) curve lies above the 45 degree line and to decreasing \( \theta \) when it lies below such that the arrows of motion shown in Figure 2 arise and the results about stability follow immediately. This completes the proof of Proposition 2.

**Derivation of (8).** The critical \( \sigma \) above which all athletes dope is obtained by setting \( A = 0 \) in (7), which provides \( \sigma = c/(\beta S_t - \phi) \equiv \bar{\sigma} \). For \( \bar{\sigma} \) to exist within \([0, 1]\) it has to be smaller or equal to unity, requiring that \( S_t \geq (c + \phi)/\beta \equiv S_{\text{high}} \). Likewise, when \( S_t < \phi \), the critical level of \( \sigma \) above which all athletes stay clean is obtained by setting \( A = 1 \) in (7), which provides

\[
\sigma = (\alpha[1 + \theta_t(\lambda - 1)] - c)/(\phi - \beta S_t) \equiv \bar{\sigma}.
\]

For \( \bar{\sigma} \) to exist within \([0, 1]\) it has to be smaller or equal to unity, requiring that \( S_t \leq \{\phi + c - \alpha[1 + \theta_t(\lambda - 1)]\}/\beta \).

Generally, integrating the area below the threshold (7) within the limit 0 and \( x \) provides
\[
\int_0^x \frac{c - \sigma(\beta S_t - \phi)}{\alpha[1 + \theta_t(\lambda - 1)]} d\sigma = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]} \left| c\sigma - \frac{\sigma^2}{2} (\beta S_t - \phi) \right|_0^x.
\]

If \( S_t > S_{\text{high}} \), integrate up to \( x = \bar{\sigma} \) to obtain

\[
1 - \theta = \frac{c^2}{2\alpha[1 + \theta_t(\lambda - 1)](\beta S_t - \phi)}.
\]

Likewise for \( S_t \) in the medium range, integrate up to \( x = 1 \) to obtain

\[
1 - \theta = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]} \left[ c - \frac{1}{2} (\beta S_t - \phi) \right].
\]

For \( S_t \leq S_{\text{low}} \) integrate up to \( x = \bar{\sigma} \) and add \( (1 - \bar{\sigma}) \). This provides

\[
1 - \theta = \frac{1}{\alpha[1 + \theta_t(\lambda - 1)]} \left[ c\sigma(\alpha[1 + \theta_t(\lambda - 1)] - c) \right] + \frac{\sigma^2}{2(\phi - \beta S_t)} + 1 - \frac{\sigma^2}{2(\phi - \beta S_t)}.
\]

Collecting terms, we get (8).

**Proof of Proposition 5.** The proof is easiest by noting that at any equilibrium \( \theta_t = S_t \) and by applying the implicit function theorem with respect to \( S_t \). Begin with taking the derivative with respect to \( S_t \)

\[
\frac{\partial G}{\partial S_t} = \begin{cases} 
\frac{c^2 \beta}{2\alpha[1 + \theta_t(\lambda - 1)](\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \\
\frac{\beta}{2\alpha[1 + \theta_t(\lambda - 1)]} & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
\frac{(\alpha[1 + \theta_t(\lambda - 1)] - c)^2 \beta}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)^2} & \text{for } S_t \leq S_{\text{low}}.
\end{cases}
\]

and notice that it is everywhere positive. Next, take the derivative with respect to \( \lambda \)

\[
\frac{\partial G}{\partial \lambda} = \begin{cases} 
\frac{2\alpha^2 \theta_t}{\alpha[1 + \theta_t(\lambda - 1)]^2 (\beta S_t - \phi)} & \text{for } S_t \geq S_{\text{high}} \\
\frac{\alpha \theta_t}{\alpha[1 + \theta_t(\lambda - 1)]^2} \cdot \left[ c - \frac{1}{2} (\beta S_t - \phi) \right] & \text{for } S_{\text{high}} \geq S_t \geq S_{\text{low}} \\
\frac{4\alpha^2 \theta_t \{\alpha[1 + \theta_t(\lambda - 1)] - c\}(1 + \theta_t(\lambda - 1)) - \alpha \theta_t(\alpha[1 + \theta_t(\lambda - 1)] - c)^2 (\phi - \beta S_t)}{2\alpha[1 + \theta_t(\lambda - 1)](\phi - \beta S_t)^2} & \text{for } S_t \leq S_{\text{low}}.
\end{cases}
\]

From this follows immediately that \( \partial G/\partial \lambda > 0 \) for \( S_t \geq S_{\text{high}} \). For the intermediate range of \( S_t \) note that here \( S_t < S_{\text{high}} \), i.e.

\[
(c + \phi)/\beta \geq S_t \implies c \geq \beta S_t - \phi \implies c > (\beta S_t - \phi)/2.
\]

Thus \( \partial G/\partial \lambda > 0 \) within the intermediate range. For \( S_t < S_{\text{low}} \), the derivative is positive if

\[
2\alpha \{\alpha[1 + \theta_t(\lambda - 1)] - c\}(1 + \theta_t(\lambda - 1) - \{\alpha[1 + \theta_t(\lambda - 1)] - c\}^2 > 0, \text{i.e. if}
\]
2\alpha [1 + \theta_t(\lambda - 1)] - \alpha [1 + \theta_t(\lambda - 1)] + c = \alpha [1 + \theta_t(\lambda - 1)] + c > 0, which is true. Thus \( \partial G / \partial \lambda > 0 \) everywhere. The final step combines the results, \( \partial S_t / \partial \lambda = - (\partial G / \partial \lambda) / (\partial G / \partial S_t) > 0 \).

The derivatives of \( \partial G / \partial c < 0, \partial G / \partial \beta > 0, \) and \( \partial G / \partial \phi < 0 \) can be immediately read off (8). Likewise \( \partial G / \partial \alpha > 0 \) for all \( S_t \geq S_{low} \). For \( S_t \leq S_{low} \) note that the sign of the derivative of \( G \) with respect to \( \alpha \) is the same as

\[
\frac{(\alpha [1 + \theta_t(\lambda - 1)] - c)^2}{\alpha} \frac{\partial}{\partial \alpha} = \frac{1}{\alpha^2} \cdot (\alpha [\theta_t(\lambda - 1)] - c) \cdot (\alpha [\theta_t(\lambda - 1)] + c) > 0.
\]

Applying the implicit function formula \( \partial S_t / \partial x = - (\partial G / \partial x) / (\partial G / \partial S_t), x = \{\alpha, \beta, c, \phi\} \) completes the proof.

**Derivation of (11).** As in the derivation of (4) the critical \( \sigma \) above which all athletes dope is obtained by setting \( A = 0 \) in (10), which provides \( \sigma = c / (\beta S_t - \phi) \equiv \bar{\sigma} \). For \( \bar{\sigma} \) to exist within \([0, 1]\) it has to be smaller or equal to unity, requiring that \( S_t \geq (c + \phi) / \beta \equiv S_{high} \). Likewise, when \( S_t < \phi \), the critical level of \( \sigma \) above which all athletes stay clean is obtained by setting \( A = 1 \) in (10), which provides \( \sigma = (z_t^\gamma - c) (\phi - \beta S_t) \equiv \bar{\sigma} \). For \( \bar{\sigma} \) to exist within \([0, 1]\) it has to be smaller or equal to unity, requiring that \( S_t \leq (c + \phi - z_t^\gamma) / \beta \).

Generally, integrating the area below the threshold (11) within the limit 0 and \( x \) provides

\[
\int_{0}^{x} \left[ \frac{c - \sigma (\beta S_t - \phi)}{z_t} \right]^{1/\gamma} d\sigma = - \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{1}{\beta S_t - \phi} \left[ (c - \sigma (\beta S_t - \phi))^{1+1/\gamma} \right]_{0}^{x}.
\]

If \( S_t > S_{high} \), integrate up to \( x = \bar{\sigma} \) to obtain

\[
1 - \theta = \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{c^{1+1/\gamma}}{\beta S_t - \phi}.
\]

Likewise for \( S_t \) in the medium range, integrate up to \( x = 1 \) to obtain

\[
1 - \theta = \frac{\gamma \left( c^{1+1/\gamma} - (c + \phi - \beta S_t)^{1+1/\gamma} \right)}{(1 + \gamma)(\beta S_t - \phi) z_t}
\]

To obtain \( 1 - \theta \) for the case \( S_t \leq S_{low} \), integrate up to \( x = \bar{\sigma} \) and add \( (1 - \bar{\sigma}) \). This provides

\[
1 - \theta = - \frac{1}{z_t} \frac{\gamma}{1 + \gamma} \frac{1}{\beta S_t - \phi} \left\{ \left[ \frac{c + z_t^\gamma - c}{\phi - \beta S_t} \right]^{1+1/\gamma} - c^{1+1/\gamma} \right\} + 1 - \frac{z_t^\gamma - c}{\phi - \beta S_t}.
\]

Noting that \( (z_t^\gamma)^{(1+\gamma)/\gamma} = z^{1+\gamma} \) and \( z_t^{1+\gamma} / z_t = z_t^\gamma \) this expression simplifies to

\[
\theta = \frac{1}{\beta S_t - \phi} \left\{ \frac{\gamma}{1 + \gamma} z_t^\gamma - z_t - \frac{c^{1+1/\gamma}}{z_t} \cdot \frac{\gamma}{1 + \gamma} + c \right\}.
\]

Collect terms to get (11).
References


