It’s a Sin – Contraceptive Use, Religious Beliefs, and Long-Run Economic Development

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Abstract. This study presents a novel theory on the interaction of social norms, fertility, education, and their joint impact on long-run economic development. The theory takes into account that sexual intercourse is utility enhancing and that the use of modern contraceptives potentially conflicts with prevailing social norms (religious beliefs). The theory motivates the existence of two steady states. At the traditional steady state, the economy stagnates, fertility is high, education is minimal, and the population sustains a norm according to which modern contraceptives are not used. At the modern steady state, the population has abandoned traditional beliefs, modern contraceptives are used, fertility is low and education and economic growth are high. Social dynamics explain why both equilibria are separated by a saddlepoint-equilibrium (a separatrix), i.e. why it is so hard to transit from the traditional regime to the modern regime.

Keywords: religion, fertility, sex, contraceptive use, education, economic growth.

JEL: 040; I25; J10; Z12.

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1. INTRODUCTION

The literature on long-run economic development has established the onset of the fertility transition as an important prerequisite for the take-off to modern growth (Galor, 2005, 2011; Dalgaard and Strulik, 2013). Fertility, in turn, depends crucially on the use of modern contraceptives. More precisely, the demographic literature identifies contraceptive use as one of the leading proximate determinants of fertility (Bongaarts and Potter, 1983). But what are the deep determinants of contraceptive use? In particular, why are modern contraceptives not used when they are available at little or no cost? One potential explanation, which is explored in this paper, is that the use of contraceptives conflicts with religious beliefs and entrenched social norms.

When contraceptives are not used and fertility is high, education levels of children are low. This is the well-known child quality-quantity trade-off at the center of unified growth theory (see Galor, 2005; 2011 for surveys). In the present context, the quantity-quality trade-off generates a further feedback at the aggregate (community- or society-) level, which makes the prevalence of traditional beliefs sustainable. Specifically, it will be assumed that the value of traditional religious beliefs is individual-specific and increasing in the aggregate number of believers, as argued in the economics of religion (e.g. Iannaccone, 1998) and declining in education (or income), capturing the impact of increasing knowledge and modernization (Inglehart and Baker, 2000).

We show that this setup motivates the existence of two locally stable steady states. At one steady state, individuals share a belief in traditional religion and do not use contraceptives. Fertility is high, education is low, and the economy stagnates. At the other steady state, traditional beliefs are abandoned and contraceptives are used. Fertility is low, education is high, and the economy grows at a constant rate. The steady states are separated by a separatrix, i.e. a threshold for which the dynamics on either side lead to one of the steady states.\(^1\)

This means that in order to leave the steady state of stagnation, society has to solve a collective action problem according to which sufficiently many individuals start using contraceptives. This observation explains the self-sustainability of the steady states and why it is sometimes so hard to instigate widespread use of contraceptives even when they are freely distributed. The size

\(^1\)We define as religion the belief in and worship of a superhuman controlling power, especially a personal God or gods. We define as religiosity the extent to which religion influences individual and societal behavior. Specifically, in our theory, religiosity is measured by the population share that adheres to traditional religion, which is a determinant of the individually experienced value of religion and individual behavior.
of the collective action needed to leave stagnation depends on the location of the separatix in the religiosity–education space. If the value of traditional religion is sufficiently low or if the productive value of education is sufficiently high, the traditional equilibrium ceases to exist and society converges towards balanced growth. Along the transition path, average fertility and the share of believers in traditional religion are gradually declining, and average education, aggregate growth, and the share of users of modern contraceptives are gradually rising.

The paper contributes to unified growth theory by emphasizing the role of education and the fertility transition for the take-off to long-run growth.\(^2\) The channel of traditional religion and contraceptive use, however, remained unexplored by conventional unified growth theory. Conventional theory ignores the human desire for sexual intercourse and thus, naturally, the use of contraceptives can play no role for the fertility transition. Implicitly, conventional theory assumes that sex is a functional and joyless activity executed to achieve a desired number of children. This approach, however, makes it difficult to discuss the role of religion and social norms for the use of contraceptives.

Here, we explicitly acknowledge that sex is a utility enhancing activity. The use of modern contraceptives allows households to experience utility from sex without a proportional increase in child births. Contraceptive use, however, may conflict with the religious beliefs of households. For linguistic ease, we define traditional religion as a belief-system, shared by members of a community, which prohibits the use of modern contraceptives. The value of traditional religion is assumed to be individual-specific and increasing in the number of households sharing these beliefs. The value of the alternative (modern) belief-system is, for simplicity and without loss of generality, normalized to zero. Households, as usual, decide on the number of children and their education. In the present context, they additionally solve the meta-decision problem of whether they share traditional beliefs and abstain from contraception or whether they abandon traditional beliefs, enjoy more sex, and limit their fertility. Since child rearing is a time consuming activity, the limitation of fertility generates extra time (potential income), which is used by households to improve their children’s education.\(^3\)


\(^3\) According to Akerlof and Kranton’s (2000) economic theory of identity choice, individuals are not necessarily aware that they are actively choosing the utility-maximizing identity. Referring to Friedman’s (1953) general methodology of positive economics it is only important that they behave as if they maximize their utility. Notice that a religious identity can be relatively easily adopted because it is less ascriptive than other dimensions of
The assumption that sex is a utility enhancing activity is perhaps self-evident but there is also compelling evidence from the happiness literature (Blanchflower and Oswald, 2004; Kahneman et al. 2004). The fact that humans like sex is readily explained by evolutionary psychology. In pre-modern times, when contraceptives were unavailable (and reproductive biology not yet understood), enjoying sex was a Darwinian fitness-maximizing trait. The notion that during evolution humans had no clear notion of how sexual intercourse was related to fertility, is helpful to motivate the desire for sex without the desire for (more) children as well as the demand for contraceptives (Wright, 1994; Potts, 1997). Acknowledging that sexual intercourse provides pleasure allows us to investigate the double role of contraceptives, namely to reduce fertility and to have more sex. The latter, obviously, cannot be explained by the alternative approach according to which sex is a joyless activity and individuals are assumed to give birth to a given number of children that can be reduced only by the use of contraceptives. Our approach, in contrast, captures the phenomenon that declining fertility is not only associated with increasing education and a reduced role of traditional religion but also with increasing sexual intercourse, in line with the historical evidence. For example, during the fertility transition in Mexico, women with secondary education reported to have 40 percent more sex than uneducated women although their fertility was less than half (Pritchett, 1994). For the U.S., Trussell and Westoff (1980) document increasing sexual intercourse in the 60s and 70s, i.e. after invention of the pill and the onset of the second demographic transition, and a positive association of coital frequency with the use of effective contraceptive methods. For the time before the mid 20th century we have to rely on historical narratives. Here, scholars agree that the arrival of new methods of birth control led to the demise of the 19th century Victorian prescriptions about continence and self-control. In the early 20th century, the writings of Sigmund Freund and Havelock Ellis pioneered the strife for sexual gratification rather than procreation in marriage and they were soon followed by many other writers. Inspired by these new ideas, marriage was increasingly approached with the expectation of erotic enjoyment (D’Emilio and Freedman, 1988, Ch. 10). The overarching impression from the literature is thus that sexual intercourse increased significantly during the 20th century, along with the decline in fertility. For contemporary developing countries, Pritchett (1994) observes from the DHS data that coital frequency is generally higher in households that use contraceptives.

identity like, for example, ethnicity. See Caselli and Coleman (2013) for a recent economic theory of ethnic identity choice.
For the developed countries of the West, economic growth during the 20th century was accompanied by a religious transition (Bruce, 2011; Paldam and Gundlach, 2013; Strulik, 2016c) with a bi-causal negative relationship between religiosity, measured by church attendance, and economic development (Herzer and Strulik, 2013). As a result we observe a synchronized timing of the fertility transition and the religious transition. The present paper argues that this may not be mere coincidence.\footnote{Of course, religion influences economic development and the fertility transition also through other channels than sexual attitudes and contraception; for example through its positive impact on the desire for savings (Cavalcanti et al., 2007) or its negative impact on the desire for pleasure from consumption (Strulik, 2016c). See Lesthaeghe and Wilson (1986) for a survey on the impact of religion on fertility through changing (family) values and norms.}

The literature provides ample evidence supporting the impact of religion and religious beliefs on fertility and contraceptive use. With respect to the historical evolution of the (mainly Christian) West, it is shown that the prevalence of Catholicism and the intensity of religious beliefs had a delaying impact on the onset of the fertility transition (Lesthaeghe and Wilson, 1986; Hacker, 1999; Adsera, 2006; Frejka and Westoff, 2008; Zhang, 2008; Baudin, 2015; see McQuillan for a survey and a discussion of potential mechanisms). In principle, a high fertility rate of believers may alternatively be explained by avoidance of contraception or by the Church’s promotion of sexual intercourse in marriage. In practice, however, there is more support for the first alternative. Traditionally Christian faith seems to be characterized by a negative attitude towards sexuality, based on the writings of Paul of Tarsus (who praised celibacy and virginity and regarded lust, even in marriage, as adultery), Augustine of Hippo (who equated sex with original sin) and many other church leaders. Instead, “the [Catholic] Church has always taught the intrinsic evil of contraception” (Pontifical Council for the Family, 1997).

In modern times, in contrast, we observe a great variety of Christian attitudes towards contraception, depending on denomination. With respect to religious beliefs across contemporaneous developing countries, Heaton (2011) finds that the historical distinctive behavior of Catholics has almost vanished. Instead, a significant difference between Christians and Muslims emerges. Christians are found to be more than twice as likely to have used contraception. Iyer (2002) finds for a sample from Southern India, that 99% of Muslim women but only 19% of Hindu women believe that contraceptive use is against their religion. Muslim women are found to be less likely to use contraceptives but the significant impact of religion disappears once socioeconomic factors (education of husband and wife) are controlled for. This led Iyer to the conclusion that it may be the way religion is interpreted by the local religious community rather than theological
doctrines that influence the use of contraceptives. Moreover, Iyer documented that there was very little interaction of women outside their own religious community. In a similar vein, Munshi and Myaux (2006) show that in rural Bangladesh the uptake of contraceptives depends crucially on the prevalence of contraceptive use in one’s religious group within the village (Muslim or Hindu), suggesting that fertility norms operate through shared religious beliefs. For Ghana, Gyimah et al. (2012) show that Muslim women and adherents of traditional African religion were more likely to have never used contraception compared with Christian women and that this significant difference does not disappear after controlling for socio-economic characteristics.

Many of the countries in which the fertility transition and the use of modern contraceptives is most severely delayed are situated in Sub Saharan Africa. The cultural roots of high African fertility most likely stem from traditional African religion emphasizing the cult of ancestors and the succession of the generation (Caldwell and Caldwell, 1987). In fact, some country studies suggest that Christian religious socialization raises the demand for contraceptives in Africa because Christian religious leaders propagate modern family planning (Agadjanian, 2001; Yeatmen and Trinitapoli, 2008). According to traditional African religion, in contrast, high fertility is associated with “divine approval and approbation by both living and dead ancestors”, whereas low fertility is “interpreted as evidence of sin and disapproval” (Caldwell and Caldwell, 1987, p. 416) and contraception is regarded as “unnecessary” and “evil” (ibid., p. 424). These aspects of traditional religion are found to be very persistent. Traditional beliefs pertaining to fertility and the use of contraceptives have survived regardless of the fact that contemporary Africans are mostly Christians or Muslims (ibid, p. 427). A recent study by the Pew Research Center (Lipka, 2014) documents that African countries are the most morally opposed to contraception. In Nigeria and Ghana roughly half of the population regarded contraceptive use as morally unacceptable, only Pakistan displayed a higher rate of opposition (for comparison, the share of the morally opposed against contraception is 7 percent in the U.S. and 1 percent in Germany).

Consequently, demographers diagnose Sub-Saharan Africa as the only region in the world where low levels of contraceptive use and high fertility still persist (Cleland, 2009). According to the most recent DHS surveys on Sub Saharan African countries, the contraceptive prevalence rate is still very low at an average of 20% and the total fertility rate is still very high at 5.0. The slow development in some Sub Saharan African countries has motivated demographic research to

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5 Contraceptive prevalence is the current use of modern contraception methods among currently married women. Data source: ICF (2012); 35 Sub-Saharan African surveys, average survey year 2007.
speculate whether in some countries the fertility transition stalled in mid-transition and whether the demand for contraceptives has leveled off (Bongaarts, 2006).

This paper is related to a small body of literature on social norms and fertility. Palivos (2001) and Bhattacharya and Chakraborty (2012) proposed models of social conformism in which the individual utility that parents derive from their children depends on the average family size in society. Palivos investigates a setup in which fertility incurs opportunity costs in terms of less parental education and shows that complementarities can motivate the existence of multiple locally stable equilibria, sustained by a high or low fertility norm. Bhattacharya and Chakraborty integrate child mortality in this setup and show that net fertility declines as a response to improving child mortality when the society is situated at a high-fertility equilibrium. The role of sex, contraceptive use, and shared traditional religion for the emergence and sustainability of equilibria is not investigated.

Baudin (2010) develops a model of a society consisting of two types of individuals, traditionalists and modernists. These two types work in different sectors, receive a distinct income, and try to sustain a distinct fertility norm via vertical socialization as in Bisin and Verdier’s (2001) model of preference formation. He then investigates how exogenous income shocks or exogenous technological progress in the two sectors affect aggregate fertility and norm sustainability. Bar-El et al. (2013) employ a partial equilibrium model of vertical socialization in which a preference for many children is identified with religiosity and use the model to derive testable hypotheses on the evolution of secularization. This paper shares with these studies a focus on the role of culture for fertility. It focuses on the role of sexual desire, contraceptive use, education, and endogenous long-run growth, which were not discussed in the earlier literature.\(^6\)

The role of contraceptives for the fertility transition is also investigated by Bhattacharya and Chakraborty (2016) and Strulik (2016a). Bhattacharya and Chakraborty also investigate (numerically) the role of social diffusion and conformism. According to their approach, long-run growth is exogenous and the fertility transition and the uptake of modern contraceptives happens regardless of the existence of social norms. Instead they focus on the speed of the historical fertility transition in England. In our paper, in contrast, social norms (specifically traditional religion) is key for a locally stable equilibrium of high fertility and economic stagnation. Once

\(^6\) Here we focus on birth control as motivation for the use of contraceptives. With respect to condoms there exists another motive, protection from disease transmission. See Greenwood et al. (2013) for a general equilibrium model focusing on sexual behavior and disease transmission (and ignoring fertility).
the fertility transition is initiated our model explains the gradual take-off to endogenous long-run
growth as well as the dissolution of traditional norms. At the conceptual level, Bhattacharya
and Chakraborty do not explicitly model the utility derived from sex as a motive to use contra-
ceptives. According to their view, without contraception individuals by assumption give birth
to a fixed number of children. Couples use contraception in order to get a smaller number of
children than that and the use of contraceptives incurs fixed and variable costs. This means
that the study does not address how the use of contraceptives allowed individuals to have more
sex such that the fertility transition is accompanied by increasing sexual intercourse and the
dissolution of traditional norms. Strulik (2016a), like this study, acknowledges the desire for sex
as a main driver of human reproductive behavior but focuses on the role of costs and efficacy
of contraceptives and, in its application, on the historical fertility transition of the West. The
study neglects the role of religion and social norms to sustain a locally stable equilibrium of high
fertility, which is in the focus of the present paper.

This paper is organized as follows. Section 2 sets up the model, computes optimal fertility and
child education under traditional and modern beliefs, solves the problem of individual identity
choice, and computes the division of society according to shared beliefs. Section 3 computes
social dynamics and determines the steady states of stagnation and long-run growth. It computes
the shifters of the separatrix of the two equilibria and identifies the return to schooling as an
important determinant for the existence of a locally stable steady state of stagnation. In the
appendix we generalize the model towards imperfect or absent intergenerational transmission
of knowledge. In this case, the former growth effects become level effects and the separatrix
separates a traditional low-income equilibrium from a modern high-income equilibrium. Section
4 concludes.

2. The Model

2.1. Households and Firms. Consider an economy populated by a large number of households
and a measure one of competitive firms. At any given time, firms use human capital as input
in order to produce output \( y_t = \bar{h}_t \ell_t \), in which \( \bar{h}_t \) is the average level of human capital in the
economy and \( \ell_t \) is aggregate labor supply. Household income is thus given by its supply of
human capital.
Households consist of couples who cooperatively maximize utility from spending income on consumption, from having children, from the education of their children, and from having sex. For better comparison with related models of one-person-households, we measure all variables in units per parent, such that \( n_t \) is the number of children per parent, \( c_t \) is consumption expenditure per parent, etc. Notice that there is no conflict of interest between spouses.\(^7\)

Furthermore, households experience utility from sharing a religious identity (or, more generally, a value system) with other households. Sharing a traditional religious value system requires avoiding the use of modern contraceptives \((u_t = 0)\). It provides a utility \( \nu_t R(i) \), in which \( \nu_t \) is a measure of the general value of traditional religion shared by all members of society and \( R(i) \) is a measure of the individual-specific utility experienced by individual \( i \). For simplicity we assume that \( R(i) \in (0, 1) \) is uniformly distributed in each generation. Notice that \( R(i) \) is individual-specific but constant over time, while \( \nu_t \) applies to all individuals and potentially varies over time. The determinants of the general value of traditional religion are exogenous to the individual household but endogenous at the community-level. Details will be introduced in Section 3. The value of the alternative religious identity (no religion, or a religion not condemning the use of modern contraceptives) is normalized to zero.\(^8\)

In order to determine an analytically tractable solution, the household utility function is assumed to be separable and logarithmic. It reads

\[
U = \log c_t + \alpha \log n_t + \gamma \log e_t + \sigma \log s_t + \chi \nu_t R(i),
\]

in which \( e_t \) is education per child, \( s_t \) is sexual activity, and \( \chi \) is an indicator variable that assumes the value of unity if the household abstains from modern contraceptives. Otherwise, \( \chi = 0 \). Besides the last two elements, the utility function is standard in unified growth theory.

The optimal individual choices depend on the preferred religious identity and thus should be

\(^7\) In an Appendix (available on request) we show that the theory can be extended towards a bargaining couple, in which females contribute less to household income and spend more time on child-rearing. This provides some interesting results on female empowerment and the uptake of contraceptives but leaves all results from the basic model intact. At the expense of rising complexity it is also conceivable to extend this approach towards separate-spheres bargaining (Doepke and Kindermann, 2016) and to take into account deviating fertility preferences of men and women (Prettner and Strulik, 2014).

\(^8\) Naturally, the results would not change if we replace the utility from sharing the traditional religion with a disutility experienced from not obeying traditional norms. The latter approach would imply a different interpretation of the results, based on social conformism (Bernheim, 1994) rather than religious identity (Akerlof and Kranton, 2000).
indexed by $i$ as well. In order to avoid notational clutter we suppress the individual index unless needed for clarification.

For simplicity we measure $s_t$ such that without the use of modern contraceptives a unit of $s_t$ implies a unit of $n_t$. This number may be thought of as already taking into account costless (and socially accepted) traditional methods of contraception, such as breastfeeding or the rhythm method. Using a unit of modern contraceptives prevents the birth of $\mu$ children. The parameter $\mu$ thus controls the effectiveness of modern contraceptives. Taking an upper limit of fertility, given by female fecundity, $\bar{n}$ into account, we thus have

$$n_t = \min \{s_t - (1 - \chi)\mu u_t, \bar{n}\},$$

in which $u_t$ is household demand for modern contraceptives. We assume that contraceptives are available at negligible costs. Of course, contraceptive use could be low as well because of limited availability or high prices (channels that have been addressed in an otherwise similar model by Strulik, 2016a). Here, the assumption of unlimited access and zero costs helps us to focus the analysis on social dynamics and attitudes and beliefs as a determinant of contraceptive use. At the dawn of the fertility transition in the West, the costs of modern contraception were significant (Guinnane, 2011), which makes them potentially crucial for the onset and speed of the fertility tradition. In contemporaneous developing countries, in contrast, contraception is frequently freely available and the question is why it is not more widely used. Moreover, the present study focuses on steady states and their stability characteristics, and not on the speed of transitional dynamics. Strulik (2016a) provides a unified growth theory in which the costs of contraceptives are crucial for the onset and speed of the historical fertility transition but irrelevant for the steady states of stagnation and long-run growth. When contraceptives are unavailable there is naturally no choice, which implies that any economy is likely to be situated originally at the traditional equilibrium where modern contraceptives are not used.\footnote{Here we implicitly assume that individuals are fecund during their whole life. Adding a third period of old (and infertile) age would allow to capture declining contraceptive use due to population aging at the end of the demographic transition as observed in some advanced economies.}

Households are endowed with human capital $h_t$ and one time unit per adult, which is spent on work, child rearing, and sex. This implies the budget constraint

$$h_t(1 - \phi n_t - e_t n_t - \tau s_t) - c_t = 0,$$
in which $\phi$ is the time spent on child rearing, $e_t$ is the time spent on education per child, and $\tau$ is the time spent on sex. The presence of a time cost for sex $\tau$ prevents sex from increasing without limit in a growing economy. An arbitrarily small time cost is sufficient to achieve asymptotically constant sexual activity. Alternatively, we could use a satiation level or a physiological upper limit for sex without changing the results.\footnote{An extended version of the model would include a preference for leisure and the time spent on leisure activities (other than sex) would compete as usual with the time spent working. Leisure time would, however, also compete with the time spent on sex such that all qualitative results of the simple model would be preserved.}

As common in unified growth theory (e.g. Galor, 2005, 2011), we make the analytically convenient assumption that education costs parental time. In the Appendix we set up an alternative version of the model according to which education is provided by teachers in a separate schooling sector and show that this leaves our results unaffected.

Households maximize (1) subject to (2)–(3), given non-negativity constraints on all variables. Additionally, we assume $\gamma < \alpha$. The assumption ensures that parents always prefer to have children although children could be avoided at no cost. If parents decide not to share the traditional religion and to use contraceptives, the solutions for consumption, fertility, education, contraceptive use, and sex are:

\begin{align*}
  c_t &= \frac{h_t}{1 + \alpha + \sigma}, \\
  n_t &= n_M \equiv \frac{\alpha - \gamma}{(1 + \alpha + \sigma)\phi}, \\
  e_t &= e_M \equiv \frac{\gamma\phi}{\alpha - \gamma}, \\
  u_t &= u_M \equiv \frac{\phi\sigma - (\alpha - \gamma)\tau}{(1 + \alpha + \sigma)\phi\mu\tau}, \\
  s_t &= s_M \equiv \frac{\sigma}{(1 + \alpha + \sigma)\tau}.
\end{align*}

A subscript $M$ identifies the solution as an optimal solution, conditional on sharing a modern belief system (as opposed to traditional religion). Notice that all outcome variables aside from consumption are constant over time and independent from parental education and income. This artifact originates from the simplifying assumption of zero costs of contraception. As shown by Strulik (2016a), introducing a (small) cost of contraception is a powerful device to rationalize that income and human capital are negatively associated with fertility and positively with the education of offspring. Here we neglect this feature in order to focus on the role of religious
norms for fertility and education. Inspection of (8) shows that even couples with modern beliefs do not necessarily use modern contraceptives, which motivates the following assumption.

**Assumption 1.** The desire for sex is sufficiently strong such that couples with modern attitudes towards contraception ($\chi = 0$) use contraception, that is, $\sigma > \bar{\sigma} \equiv (\alpha - \gamma)\tau/\phi$.

The threshold for the use of modern contraceptives is increasing in the desire for having children and the time cost of having sex, and it is declining in the desire for education and child rearing costs. These results are very intuitive. In the context of the present paper, a solution according to which modern contraceptives are never used is uninteresting. Assumption 1 focuses the analysis on an interesting problem.

If parents share traditional beliefs and maximize (1) subject to (2)–(3), given $\chi = 1$, they arrive at the optimal solution

\[
n_t = n_T = \frac{\alpha - \gamma + \sigma}{(1 + \alpha + \sigma)(\phi + \tau)},
\]

\[
e_t = e_T = \frac{\gamma(\phi + \tau)}{\alpha - \gamma + \sigma},
\]

\[
s_t = s_T = \frac{\alpha - \gamma + \sigma}{(1 + \alpha + \sigma)(\phi + \tau)},
\]

and $c_t$ as in (4). An index $T$ indicates the optimal solution contingent on sharing traditional beliefs. Comparing the traditional and the modern equilibrium, we find:

**Proposition 1.** Couples sharing modern beliefs have fewer and better educated children and have more sex than couples sharing traditional beliefs.

For the proof, we use the fact that at the modern equilibrium from Assumption 1, $\sigma > \bar{\sigma}$, implying $(\alpha - \gamma)\phi + \sigma \phi > (\alpha - \gamma)\phi + (\alpha - \gamma)\phi$, that is, $\phi(\alpha + \gamma + \sigma) > (\phi + \tau)(\alpha - \gamma)$, such that $e_M > e_T$. The proofs for fertility and sexual activity are analogous.

Abandoning traditional beliefs thus goes hand in hand with sexual liberation. Notice the essentially “non-Darwinian” character of this result. The availability and use of contraceptives offers humans the possibility to have simultaneously more sex and fewer offspring, an option unavailable to (other) animals. Lower fertility reduces child rearing time and the extra time (or, more generally, the extra potential income) is used optimally in order to improve education of children. This is “the lever of riches” (Mokyr, 1990) provided by birth control technology.
Contraceptive technology also explains why the model predicts a different response of fertility to an increasing sex drive in traditional and modern societies:

**PROPOSITION 2.** An increase in the desire for sex $\sigma$ increases fertility at the traditional equilibrium and decreases fertility at the modern equilibrium.

The proof follows immediately from inspecting equations (9) and (12). The rise of fertility at the traditional equilibrium is the expected (strong) Darwinian response to an increasing sex drive. At the modern equilibrium, this mechanism does not apply any longer. Instead, there is a (weak) non-Darwinian response because more sex demands more time, which reduces the total time available for work or child-rearing and has a second order effect on fertility through the opportunity costs of child rearing. The effect is of second order because the time cost of sex is presumably small compared to the time spent on working and child-rearing.

2.2. Contraception and Desired Fertility. In an influential paper, Pritchett (1994) documents a high (almost one-to-one) correspondence between fertility and desired fertility and shows that desired fertility is largely independent from cost and access of contraception. From these results he concludes – in contrast to the standard demographers’ view – that contraception is not an important causal or independent determinant of fertility (p. 15). We think that our model provides one way to reconcile these only seemingly contradictory literatures. Specifically, we show that contraception plays an important causal role for fertility although it is perfectly aligned with desired fertility. The reason is that religious beliefs (or, more generally, social norms) determine whether modern contraceptives are used. Comparing the modern solution (4)–(8) with the traditional solution (9)–(11) we see that desired fertility is high when religious believes prevent the use of modern contraceptives. Notice that, according to the model, fertility and desired fertility are – by assumption – perfectly aligned since there are no costs and no barriers of access to contraception. This means that our model suggests that in a reduced form regression, actual fertility is perfectly explained by desired fertility and that adding contraceptive prevalence on top would contribute nothing (which is close to the Pritchett finding). However, contraceptives and the religious motives for (not) using them are an explanatory variable (an instrument) for desired fertility such that the inferior use of contraceptives causally explains high fertility and stalled demographic transition (which is close to the demographers’ view).
The model may also be helpful to shed new light on the “unmet needs” debate (Pritchett, 1994; Casterline and Sinding, 2000). Unmet needs refer to the condition of wanting to avoid or postpone childbearing while not using contraception. For an assessment it is important to notice that women (or men or couples) are not directly asked whether an unmet need for contraception exists. Instead unmet needs are reconstructed by analysts from separate questions on fertility desire and contraceptive use. In the present context this means that it is crucial whether individuals internalize when answering the questions that they are constrained by social norms (a phenomenon addressed also as competing preferences; Casterline and Sinding, 2000).

Specifically, a women sharing traditional religion who has already given birth to \( n^M \) or more children may state that she does not want any more children if she is not internalizing that she is bounded by religious norms and the desire for sex. At the same time she reports the non-use of contraception as proscribed by traditional religion. Taking sexual desire and the religious norm of abstention from modern contraception into account she bears actually \( n^T > n^M \) children and the analyst observes an “unmet need”. This “unmet need”, however, is not an “unmet demand” since (non-) demand is uniquely explained by her adherence to religious norms rather than the cost of or access to contraceptives. On the other hand, a religious women internalizing the constraints of sexual desire and religious norms when answering the interviewers question would report that she desires \( n^T \) children and the analyst would observe no “unmet need”. Regardless of whether an “unmet need” is diagnosed or not, the non-use of contraception causally explains high fertility and low education and lowering costs or improving access to contraception are ineffective in changing the situation. Instead, as shown below, increasing the return to education is a powerful mechanism to initiate contraceptive use and to escape from the high fertility equilibrium.

2.3. Identity Choice. Inserting the solutions provided by (4)–(11) into (1), we get indirect utility conditioned on identity choice. Following Akerlof and Kranton (2000), we assume that individuals choose an identity that maximizes their utility. Comparing the utility differential \( U_M - U_T \), we find that individual \( i \) prefers to share traditional beliefs if

\[
R(i) \geq \bar{R}_t \equiv \omega/v_t,
\]

with \( \omega \equiv (\alpha - \gamma + \sigma) \log(\tau + \phi) - (\alpha - \gamma + \sigma) \log(\alpha - \gamma + \sigma) + (\alpha - \gamma) \log(\alpha - \gamma) - (\alpha - \gamma) \log(\phi) - \sigma \log(\tau) + \sigma \log(\sigma). \)
Proposition 3. Increasing sexual desire $\sigma$ raises the threshold to share traditional beliefs $\bar{R}_t$, $\partial \omega / \partial \sigma > 0$.

The proof computes the derivative of $\omega$ with respect to $\sigma$ as $\log \left[ \sigma (\tau + \phi) / ((\alpha - \gamma + \sigma) \tau) \right]$. The expression is positive if the term in square brackets is larger than 1, which requires $\sigma (\tau + \phi) > (\alpha - \gamma + \sigma) \tau$. This inequality is fulfilled for $\sigma > (\alpha - \gamma) \tau / \phi$, that is, under Assumption 1.

The threshold $\bar{R}_t$ separates individuals sharing traditional religion (traditional beliefs) from those sharing modern beliefs. Now, the normalization of the support of religious identity $R(i) \in (0,1)$ comes in handy since we can immediately infer population shares, taking into account potential corner solutions:

Proposition 4. The population share of individuals sharing modern beliefs (i.e. using contraceptives) is given by

$$m_t = \max \{0, \min (\omega / \nu_t, 1)\}.$$ (13)

Notice that Proposition 3 ensures that we can always find a value for $\sigma$ for which $\omega$ is strictly positive. This means that an appropriate choice of the desire for sex $\sigma$ guarantees that there exists a situation in which at least some individuals prefer to abandon the traditional identity such that the uptake of contraceptives and the take-off to modern growth become feasible.

2.4. Human Capital. Human capital of the offspring of a household of type $j$ is produced according to the production function

$$h_{t+1, j} = h_0 + A \bar{h}_t e_j,$$ (14)

in which $j = T, M$ indicates whether the household shares a traditional or a modern identity and $\bar{h}_t$ is the average human capital in society. Average human capital is thought to capture the state of knowledge in society, i.e. the knowledge that can be transferred through education to the next generation. The constant $h_0$ captures basic skills picked up without investment effort by observing parents and peers at work. The parameter $A$ is the marginal return to education per unit of human capital of the current (teacher-) generation, $A = \partial h_{t+1} / (\partial e_j \bar{h}_t)$. Here, we treat the return on education as a given parameter but it could be endogenously explained by skill-biased technological progress or structural change in a more complex unified growth model (Galor and Weil, 2000; Galor and Mountford, 2008). The linearity of (14) in $e_j$ is innocuous and could be avoided without loss of generality. The linearity in human capital is essential for
the model to create perpetual growth. See Strulik et al. (2013) for a motivation of the linearity assumption. In Appendix A, we abandon the linearity assumption and discuss the case with decreasing or absent intergenerational transmission of knowledge.

2.5. The Value of Traditional Religion. At the macroeconomic level, the value of traditional religion has social and intellectual determinants. In terms of social determinants, we assume that the value of religion this period, \( v_t \), is increasing in last period’s population share of believers, which means it is declining in last period’s share of non-believers, \( \partial v_t / \partial m_{t-1} < 0 \). This intergenerational transmission of religious values avoids strategic complementarities, which could arise if \( v_t \) were dependent on the current generation’s share of believers. Our modeling via intergenerational transmission appears to be a more plausible approach, since it allows us to capture entrenched values, which change only gradually. Formally, religious beliefs are a pre-determined variable according to the present approach, whereas they are a jump variable in models of strategic complementarities. A positive effect of the share of fellow believers on the value of religion is a standard assumption in the economics of religion (e.g. Iannaccone, 1998). Positive externalities arising from sharing one’s beliefs with others are generally considered to be a central element of any religion. An exception, which we ignore henceforth, is the formation of religious sects, for which there exists an interior optimal size of members (Iannaccone, 1998).

In terms of intellectual determinants we assume that the value of traditional religion is declining in the average level of human capital in society \( \bar{h}_{t-1} \). This is meant to capture the standard of knowledge, i.e. the degree to which the population has access to books, newspapers, etc. containing (scientific) information that questions traditional beliefs. Human capital, which coincides with income per capita in the present study, is commonly regarded as a driver of modernization and disassociation with traditional values. Inglehart and Baker (2000) investigate modernization across countries quantitatively by, among other things, computing a secular value index from the World Value Surveys. Two out of five items of the secular value index directly measure religious values. The secular value index is then found to be significantly associated with income per capita across countries. Herzer and Strulik (2013) investigate the time series properties of church attendance rates and income per capita for a panel of countries over the time span 1930-1990 and find a strong negative association with causality running in both directions. This finding is consistent with our approach according to which the causality running from church attendance (as a measure of the strength of beliefs) to income growth is
endogenously explained. The reverse causality, running from income (or human capital) to religious beliefs, is exogenous. An extensive discussion of potential channels is provided by Bruce (2011).

In principle, our results hold for any positive value function $v_t = v(m_{t-1}, h_{t-1})$ that is monotonously decreasing in its arguments and asymptotically approaching zero for perpetually increasing $\bar{h}_t$. However, it alleviates the derivation of an explicit solution to assume a Cobb-Douglas form:

$$v_t = \frac{\psi}{m_{t-1}h_{t-1}^{\epsilon}},$$

in which $\psi > 0$ measures other (constant or only occasionally varying) determinants of religious beliefs and $\epsilon$ measures the relative importance of human capital. This completes the description of the model.

3. Social Dynamics and Long-Run Economic Development

Human capital of children of parents with modern beliefs is given by $h_0 + \alpha e_M \bar{h}_t$, whereas children of traditional households have human capital $h_0 + \alpha e_T \bar{h}_t$. Thus, average human capital in society evolves according to

$$\bar{h}_{t+1} = h_0 + \frac{m_t n_m e_M + (1-m_t)n_T e_T \bar{h}_t}{m_t n_M + (1-m_t)n_T} \alpha e_M \bar{h}_t = h_0 + \frac{\phi(\phi + \tau) \gamma A \bar{h}_t}{\phi(\alpha - \gamma + \sigma - \sigma m_t) + m_t \tau(\alpha - \gamma)},$$

(16)

Forwarding (15) one period and substituting $v_t$ in (13) provides

$$m_{t+1} = \max \left\{ 0, \min \left( \frac{\omega}{\psi} m_t h_t^{\epsilon}, 1 \right) \right\}.$$  

(17)

The dynamic system (16)-(17) fully describes the evolution of the economy. It is best discussed in a phase diagram. The isocline along which human capital is constant ($\Delta \bar{h}_t = 0$) is obtained from (16) as

$$\bar{h}_t = h_0 \left[ 1 - \frac{\phi(\phi + \tau) \gamma A}{\phi(\alpha - \gamma + \sigma - \sigma m_t) + m_t \tau(\alpha - \gamma)} \right]^{-1}.$$  

(18)

In an $m_t-\bar{h}_t$ diagram, the $\Delta \bar{h}_t = 0$–isocline originates at $m_t = 0$ and $h_t = h_0/(1 - e_T A)$ and has a convex shape. For $m_t = 1$, the right-hand side of (18) simplifies to $h_0(\alpha - \gamma) / [\alpha - \gamma(1 + \phi - A)] = h_0/(1 - e_M A)$. In this section, we focus on the case in which long-run economic growth is feasible. This implies that $e_M A > 1$ (see below). This in turn implies that the isocline exhibits a pole at some $m_t < 1$. To the right of the $\Delta \bar{h}_t = 0$–isocline, i.e. for larger $m_t$, $h_t$ is increasing and to the left of the $\Delta \bar{h}_t = 0$–isocline, $h_t$ is declining.
The isocline along which the population share of individuals with a modern identity is constant is obtained from (17) as
\[
\bar{h}_t = \left(\frac{\psi}{\omega}\right)^{1/\epsilon}.
\] (19)

The $\Delta m_t = 0$–isocline is a horizontal line. The arrows of motion point towards lower $m_t$ below the isocline and towards higher $m_t$ above it.

For $h_0/(1 - eTA) < (\psi/\omega)^{1/\epsilon}$, there exists a unique intersection of the isoclines identifying the unique interior steady state $m^*, \bar{h}^*$. The phase diagram for this case is shown in Figure 1. Inspecting the arrows of motion leads to the conclusion that the interior steady state is a saddle point. These observations verify the following proposition.

**PROPOSITION 5.** For sufficiently small returns to education, i.e. for $Ae < 1 - h_0/(\psi/\omega)^{1/\epsilon}$, the economy exhibits a unique interior equilibrium, situated at $(m^*, \bar{h}^*)$, which is a saddle point. Economies starting above the stable manifold $g(m_t, \bar{h}_t)$ converge towards perpetual growth and generally shared modern beliefs, $\bar{h}_t \to \infty$, $m_t \to 1$. Economies starting below the stable manifold converge to subsistence and generally shared traditional beliefs, $\bar{h}_t \to h_0/(1 - eTA)$, $m_t \to 0$.

Aside from the possibilities stated in Proposition 5, there exists the possibility that the economy starts exactly on the stable manifold and converges towards the interior steady state. However, in contrast to infinite horizon optimal control problems, there exists no transversality condition that would move the economy on the stable manifold. This means that the economy is situated on the manifold with zero probability and diverges from it for any perturbation.

The stable manifold, as visualized in Figure 1, creates a boundary, a “separatrix” that separates the growth regime from the subsistence regime. The existence of the separatrix explains why it frequently appears to be so difficult to get rid of traditional beliefs and to initiate the use of contraceptives and the onset of the fertility transition. Consider a subsistence economy at $(0, h_0)$. In order to move out of the traditional steady state, society needs to solve a collective action problem that moves $m_t$ and $\bar{h}_t$ up beyond the separatrix. However, without any coordination device to initiate collective action, everybody would stick to the traditional religion, refrain from using contraceptives, and provide low education to their children. As a consequence, the economy stagnates. The shared belief that contraception is bad is self-sustained by the poorly educated society. Without collective action, an economy may escape from the equilibrium of
stagnation by a sufficiently large parameter change that renders the equilibrium non-existent (see below).

If instead a society surpasses the boundary (starts above the separatrix), \( \bar{h}_t \) is perpetually increasing. If the return to education \( (A) \) is small, growth eventually peters out. If \( A \) is sufficiently large, such that \( Ae_M > 1 \), as assumed above, the economy converges towards perpetual growth.

**Proposition 6.** For \( e_M A > 1 \), that is, for \( A > (\alpha - \gamma)/(\gamma \phi) \), an economy starting above the separatix \( g(h_t, m_t) \) converges towards perpetual growth at gross rate

\[
\frac{h_{t+1}}{h_t} = g \equiv \frac{\gamma \phi A}{\alpha - \gamma}.
\]

Along the transition path to constant growth,

- the share of people abandoning traditional beliefs and using contraceptives increases;
- the average number of children per woman declines;
- the frequency of sexual intercourse rises
- the share of people with high education increases; and
- the growth rate of the economy increases gradually towards \( g \).
The proof begins with noting from (17) that $m_t$ approaches unity along the growth path and thus individual human capital $h_t$ approaches average human capital $\bar{h}_t$. Inserting education $e_M$ from (7) into (16) with $m_t = 1$ provides $g$. Applying Proposition 1 and noting that an increasing share of individuals prefers the modern solution of the individual decision problem proves the remainder of Proposition 6.

**Corollary 1.** If the return to education is sufficiently large, i.e. if $A e_T > 1 - h_0 / (\psi / \omega)^{1/\epsilon}$, there exists no equilibrium of stagnation. The economy converges globally to balanced growth.

For the proof, notice that a sufficiently large return to education means that diagrammatically, the $\Delta \bar{h}_t = 0$–isocline originates above the $\Delta m_t = 0$–isocline. The associated phase diagram is shown in Figure 2.

The result is intuitive. When education is sufficiently productive, a relatively small education investment of parents suffices to let human capital grow. Perpetual growth of human capital diminishes the value of traditional religion such that more and more people abandon it and reduce their fertility and increase education. As a consequence, growth is perpetually increasing until it reaches its steady-state rate.

Considering an economy situated at or close to the equilibrium of stagnation, a sufficiently large increase in the return on education renders the equilibrium non-existent and initiates the
take-off to long-run growth. In this sense, the supply of better education (construction of local schools, better teaching materials, etc.) may well be the best population policy. Aside from deliberate schooling policy, a higher return on education may materialize through the arrival of new technology or technological knowledge through globalization (Lindner and Strulik, 2014). While a higher return to education is perhaps the most obvious parameter change initiating the escape from traditional equilibrium, other parametric shifts may be conducive to growth as well. To see this, we insert $e_T$ from (10) into the condition of Corollary 1, which then reads

$$\frac{\gamma(\phi + \tau)A}{\alpha - \gamma + \sigma} > 1 - \frac{h_0}{(\psi/\omega)^{1/\varepsilon}}. \quad (20)$$

Inspection of the right hand side of condition (20) shows that the equilibrium of stagnation disappears, ceteris paribus, when the general value of traditional religion $\psi$ is sufficiently low or when the shift parameter of the individual-specific threshold to prefer traditional religion $\omega$ is sufficiently high. Both results are, of course, hard to exploit by standard economic policy. In the European context, Enlightenment may have been such a downward shifter in the value of religion which initiated the earliest fertility decline in France and the U.S. (Lesthaeghe, 1983).

With respect to the left hand side of condition (20) we conclude that, ceteris paribus, the equilibrium of stagnation ceases to exist if the desire for education ($\gamma$) is sufficiently large, the desire for children ($\alpha$) is sufficiently low, or if the desire for sex ($\sigma$) is sufficiently large. The latter result is perhaps most interesting. It is certainly a non-Darwinian result that the high fertility equilibrium disappears when people have a strong desire for sex. This conclusion could not have been drawn in pre-modern human history when cheap and effective contraceptives were unavailable.

So far, the theory seems to support either stagnation before the onset of the fertility transition or an ongoing fertility transition and convergence towards low fertility and high growth. In order to address the observation that in some countries the fertility transition stalled halfway and the uptake of modern contraceptives has leveled off (Bongaarts, 2006) it is helpful to recall the finding that human interaction and thus the sharing of local norms happens to large degree within religious groups (Iyer, 2002; Munshi and Myuax, 2006). In the context of our model this means that each local group follows their own dynamics as represented by Figure 1 or 2. At the aggregate level of a religiously diverse country it is thus possible that some groups (e.g. those of Christian faith or non-believers) have initiated the fertility transition and moved largely forward.
towards the equilibrium at $m = 1$ in Figure 2, while other groups (e.g. Muslims or adherents of traditional African religion) are stuck at or close to the equilibrium at $m = 0$ in Figure 1. From the aggregated country-level perspective one then gets the impression that the transition leveled off (halfway, if groups are symmetric).

4. Conclusion

In this paper we proposed a theory of economic growth and stagnation based on the use of contraceptives and its impact on fertility and education. We explicitly took into account that sex is a utility enhancing activity and thus creates a natural demand for contraceptives. This allowed us to address the deep determinants of perhaps the most important proximate determinant of the delayed or even stalled fertility transition in least developed countries, that is the demand for contraceptives. Inspired by Caldwell and Caldwell (1987), we assumed that the use of contraceptives requires the abandonment of traditional religion, i.e. the sharing of social norms that condemn the use of contraceptives. We have then shown that the assumption that the value of religion is high when it is shared by many and when average human capital is low creates a self-sustained equilibrium of high fertility, low education, and economic stagnation.

If returns to education are high enough (or the value of traditional religion is low enough) the equilibrium of stagnation ceases to exist, a fertility transition is initiated, the society gradually abandons traditional beliefs, education increases, and the economy converges to balanced growth. If the intergenerational transfer of knowledge is less than perfect, there is no endogenous growth; instead the economy converges towards a higher level of income per capita. It is straightforward to embed the theory into a unified growth context. In such a setting population growth would gradually increase the knowledge frontier (as in Galor and Weil, 2000) and the local equilibrium of stagnation would become a quasi steady state, which eventually loses its basin of attraction through ongoing technological change. Traditional religion would then not prevent but “only” delay the fertility transition and the take-off to growth.

Future extensions of the theory could try to integrate a more deeply micro-founded religious behavior through efforts of parents to instill religious values in their children (vertical socialization). Further realism could be gained by replacing the unitarian household by a bargaining approach and by considering gender-specific labor supply and demand for religion. A bargaining approach could also take into account gender-specific preferences for sexual intercourse (as in
Strulik, 2016b) and discuss the role of mutual consent for sex (as in Doepke and Kindermann, 2016, with respect to mutual consent for fertility). Moreover, a multi-community version of the model could investigate whether exposure to a more modern culture of out-groups could erode the value of traditional religion and initiate the fertility transition and the escape from stagnation. Finally, it could be interesting to integrate the present approach with the health literature on contraceptive use in terms of condoms. Religious beliefs could provide an explanation for why condoms are not used although they reduce the risk of sexual disease transmission. Increasing health risk from unprotected sex in turn could motivate the abandoning of traditional beliefs and initiate the use of modern contraceptives and the fertility transition.
Appendix A. Level Effects

In this section we show that the main mechanism holds irrespective of whether the economy is capable of long-run growth. Without long-run growth, crossing the separatrix leads to a take-off from subsistence to a higher level of aggregate human capital and income per capita. In order to show this, we generalize the human capital production function to

\[ h_{t+1} = h_0 + A \bar{h}_t^\lambda e_t, \quad \lambda \in [0, 1]. \]

Moreover, we generalize the religious value function to

\[ v_t = \psi/(m_t^\nu h_{t-1}^\epsilon), \quad \nu \geq 1. \]

The dynamic system (16) and (17) is thus replaced by:

\[ \bar{h}_{t+1} = h_0 + \frac{\phi(\phi + \tau)\gamma A\bar{h}_t^\lambda}{\phi(\alpha - \gamma + \sigma - \sigma m_t) + m_t \tau(\alpha - \gamma)}, \quad (21) \]

\[ m_{t+1} = \frac{\omega}{\psi} m_t^\nu h_t^\epsilon. \quad (22) \]

For \( \nu > 1 \), there are “increasing returns” in the value of beliefs. The value of an identity increases more than linearly in the share of community members who preferred this identity in the last period.

The \( \Delta m_t = 0 \)-isocline is given by \( h_t = (\psi/\omega)^{1/\epsilon} m_t^{-(\nu-1)/\epsilon} \). For \( \nu > 1 \) the isocline is a hyperbola in the positive quadrant. For \( \nu = 1 \), it collapses to the vertical line of the last section.

We observe \( \Delta m_t > 0 \) above the isocline (for larger \( h_t \)) and \( \Delta m_t < 0 \) below. The \( \Delta h_t = 0 \)-isocline is implicitly given by

\[ G(m_t, h_t) = \bar{h}_t - h_0 - \frac{\phi(\phi + \tau)\gamma A\bar{h}_t^\lambda}{\phi(\alpha - \gamma + \sigma - \sigma m_t) + m_t \tau(\alpha - \gamma)} = 0. \]

The derivative \( \partial G/\partial m_t \) is equal to \( (\alpha - \gamma)\tau - \phi \sigma \) times a positive term. It is thus negative and finite for \( \phi \sigma > (\alpha - \gamma)\tau \), which is fulfilled due to Assumption 1. Moreover,

\[ \frac{\partial G}{\partial h_t} = 1 - \lambda \left( 1 - \frac{h_0}{h_t} \right) \geq 0. \quad (23) \]

The expression is non-negative since \( h_t \geq h_0 \) and \( \lambda \in [0, 1] \). Together this means that the slope of the \( \Delta h_t = 0 \) isocline, \( dh_t/dm_t = -\partial G/\partial m_t)/(\partial G/\partial h_t) \), is positive. The crucial difference between growth and level effects can be seen for \( h_t \to \infty \): for \( \lambda = 1 \) we observe \( \partial G/\partial h_t = 0 \) and thus \( dh_t/dm_t = \infty \). The isocline exhibits a pole, a feature that allows for long-run growth (see last section). For \( \lambda < 1 \), \( \partial G/\partial h_t > 0 \) and the slope of the isocline \( dh_t/dm_t \) is finite everywhere. This means that it eventually intersects the \( m_t = 1 \)-line. This feature inhibits long-run growth.
The isocline originates at some positive $\hat{h}_t \in (0, h_0/(1 - Ae_T))$. To the right of the isocline we observe $\Delta h_t > 0$ and to the left, we observe $\Delta h_t < 0$.

Figure 3 shows the implied phase diagram. As before, the arrows of motion identify the interior equilibrium as a saddlepoint and the associated stable manifold creates a separatrix for the traditional steady state and the modern steady state. Compared to the case from the last section, two distinct features emerge: 1. The interior steady state exists always when there are increasing returns in the share of believers. Diagrammatically, there is always an intersection of isoclines when the $\Delta m_t$–isocline is a hyperbola. Thus, a locally stable traditional equilibrium always exists under increasing returns. 2. There is no long-run growth when $\lambda < 1$. Diagrammatically, the $\Delta h_t = 0$–curve ceases to have a pole. Since the isocline is monotonously increasing and $m_t$ is bounded from above by 1, the steady state of the modern economy is obtained where the isocline intersects the $m_t = 1$–line, i.e. at $(1, \bar{h}^*)$ in Figure 3.

This result is robust to the limiting case, in which there is no intergenerational transmission of knowledge, i.e. for $\lambda = 0$. In this case, the $\Delta h_t = 0$–isocline is a positively sloped straight line and the modern steady state can be obtained analytically as $(1, h_0 + Ae_M)$. Notice that for all $\lambda \in [0, 1)$, aggregate human capital $\bar{h}_t$ and income per capita is higher at the modern steady state than at the traditional steady state. The growth effects of Section 3 become level effects.
By placing the present environment into an R&D-driven growth model a la Romer (1990) the
growth effects would re-appear. Since human capital is an essential input in R&D, growth
would be higher at the modern equilibrium and lower or absent at the traditional equilibrium.
Funke and Strulik (2000) and Strulik et al. (2013) propose innovation-based growth models in
which there is no R&D but slow growth through learning-by-doing or through physical capital
accumulation if the supply of human capital is sufficiently low.

APPENDIX B. FORMAL SCHOOLING

This section shows the equivalence of the results for in-house schooling by the parents themselves
and outsourced formal schooling by teachers financed by education expenditures of households.
In doing so, we provide the rationale for the short-cut modeling we used in the main text. The
utility function is still given by

\[ U = \log c_t + \alpha \log n_t + \gamma \log e_t + \sigma \log s_t + \chi v_t R(i). \]  

(24)

The budget constraint has to be modified to take into account that education of children is
bought on the market rather than produced at home. The new budget constraint is

\[ h_{t,i}(1 - \phi n_t - \tau s_t) = e_t n_t + c_t, \]  

(25)
in which the left-hand side comprises household income of supplying the optimal amount of time
(net of child-care and sex) on the labor market, while the right hand side comprises household
expenditures for consumption and the education of each child. The solution of the optimization
problem for households with modern beliefs is given by Equations (5), (6), (8), (9), and the
modified Equation (7)

\[ e_{t,M} = \frac{\gamma \phi h_{t,M}}{\alpha - \gamma}. \]  

(26)

Total education expenditures for households with modern beliefs are then given by

\[ m_t n_{t,M} e_{t,M} = \frac{\gamma m h_{t,M}}{1 + \alpha + \sigma}. \]  

(27)
The solution of the optimization problem for households with traditional beliefs is given by Equations (5), (10), (12), \( \chi = 1 \), and the modified Equation (11)

\[
e_{t,T} = \frac{\gamma (\phi + \tau) h_{t,T}}{\alpha - \gamma + \sigma}, \tag{28}
\]

Total education expenditures for households with traditional beliefs are then given by

\[
(1 - m_t)n_t,T e_{t,T} = \frac{\gamma(1 - m)h_{t,T}}{1 + \alpha + \sigma}. \tag{29}
\]

Recall that average human capital is denoted by \( \bar{h}_t = m_t h_{t,M} + (1 - m_t) h_{t,T} \). Furthermore, we denote the number of teachers by \( P_{t,M} \) and \( P_{t,T} \) for households with modern beliefs and households with traditional beliefs, respectively. Here we assume that teachers are endowed with average human capital and that all pupils have access to the same teacher quality. The costs of education are thus pinned down to \( \bar{h}_t P_{t,M} \) for households with modern beliefs, while the costs of education for households with traditional beliefs are given by \( \bar{h}_t P_{t,T} \). Market clearing for education implies

\[
\bar{h}_t P_{t,M} = \frac{\gamma m_t h_{t,M}}{1 + \alpha + \sigma} \iff P_{t,M} = \frac{\gamma m_t h_{t,M}}{(1 + \alpha + \sigma)h_t}, \tag{30}
\]

\[
\bar{h}_t P_{t,T} = \frac{\gamma(1 - m_t)h_{t,T}}{1 + \alpha + \sigma} \iff P_{t,T} = \frac{\gamma(1 - m)h_{t,T}}{(1 + \alpha + \sigma)h_t}. \tag{31}
\]

Assuming that average human capital of the next generation is given by basic skills \( (h_0) \) plus the productivity of teachers \( (A\bar{h}_t) \) multiplied by the number of teachers per pupil \( (P_{t,i}/n_{t,i}) \) for \( i = M,T \), leads to the evolution of human capital for households with modern beliefs and traditional beliefs as, respectively,

\[
h_{t+1,M} = h_0 + \frac{A\gamma \phi h_{t,M}}{\alpha - \gamma}, \tag{32}
\]

\[
h_{t+1,T} = h_0 + \frac{A\gamma(\tau + \phi)h_{t,T}}{\alpha - \gamma + \sigma}. \tag{33}
\]

Finally, by definition, average human capital at time \( t + 1 \) is given by

\[
\bar{h}_{t+1} = \frac{m_t n_{t,M} h_{t+1,M} + (1 - m_t) n_{t,T} h_{t+1,T}}{m_t n_{t,M} + (1 - m_t) n_{t,T}}
= \frac{A\gamma \phi (\tau + \phi) h_t}{\phi (\alpha - \gamma - m_t \sigma + \sigma) + m_t \tau (\alpha - \gamma)} + h_0. \tag{34}
\]
This expression is the same as Equation (16) in the main part which establishes the equivalence between schooling at home and schooling bought on the market.


