Long-Run Economic Growth Despite Population Decline

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Abstract. When economic growth is driven by the discovery of new ideas and physical labor is the input in ideas production, population decline inevitably leads to the stagnation of living standards. Here, I reconsider the problem of declining population when ideas are produced by an educated workforce and show that steady positive growth of ideas and income is a plausible outcome. The reason is that the accumulation of human capital offsets the negative effects of population growth. In a general equilibrium model with diminishing returns to education, I show that households generate the constant education effort needed to sustain high economic growth.

Keywords: long-run growth, innovation, fertility, education, demographic transition.
JEL: J11, J13, O11, O41.

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1. Introduction

In this paper, I reconsider the view derived from conventional ideas-driven growth models that negative population growth will lead to long-term stagnation in living standards. Negative population growth is a problem of practical relevance since the total fertility rate (TFR) is below replacement level in Europe and North America, in Japan, China, India, and basically every country that could contribute to ideas (or R&D)-driven economic growth (UN, 2022a). Replacement fertility is the level of the total fertility rate at which the population is stationary under the assumption of constant mortality rates and zero net migration. In developed countries, the replacement level is at 2.1 children per women (UN, 2022b). Sub-replacement fertility is not a new phenomenon. The TFR fell below replacement level in Europe in 1972 and in the UN country group of 'more developed regions' in 1974. In the U.S. it fell below replacement level in 1973 and briefly recovered in the years 2005–2007 after which it fell back to a current level of 1.66. The medium scenario of the UN population projections assumes that all countries approach sub-replacement fertility, which will be reached globally in 2055, after which the global TFR will converge to 1.84 in the year 2100 (UN, 2022a).

Demographic decline will affect economic performance through a variety of channels. To understand why scholars are particularly concerned about R&D-driven economic growth, recall the conventional production function for new ideas $\mathrm{d}A/\mathrm{d}t = A^{\phi}L^{\lambda}$, in which A are existing ideas that improved the quality and/or variety of goods, L is the number of people engaged in research, and $\mathrm{d}A/\mathrm{d}t$ are the new ideas produced during a time increment (e.g. a year). In equilibrium, a constant share of the population is engaged with research. The special case of $\phi = 1$ thus requires a constant population in order to generate a constant growth rate of ideas (as assumed, for example, in Romer, 1990). For the general case of $\phi < 1$, a growing population is needed to offset the diminishing returns in research (as, for example, in Jones, 1995). In both cases, the growth rate of ideas converges to zero if the population declines. Since a steady growth of ideas is essential for a steady growth of productivity and income per capita, long-term stagnation is inevitable as population declines. This profound implication has recently been emphasized in Jones (2022).

In this paper, I argue that the pessimistic conclusions about future economic growth rest crucially on the assumption that raw (or physical) labor is the human input in research. I consider a refinement of the model in which human capital, defined as individual knowledge acquired through education, is the essential input in research and show that long-term growth in ideas and per capita income can be sustained as the population declines. The reason is that diminishing returns in research are offset by increasing human capital (knowledge) of researchers and that aggregate human capital can grow as the raw labor force declines. I show that this optimistic conclusion does not require perpetually increasing effort in education nor constant returns to education. In fact, it holds when individuals spend a constant fraction of time and income on education and for (steeply) declining returns to education. The intuition is that the *value* of knowledge acquired through constant education efforts can increase over time. For example, the knowledge acquired through medical studies can increase from one generation of students to the next, despite constant study efforts.

In fact, it seems *easier* to envision long-term growth when it is ultimately driven by a meta-physical entity (like human capital) rather than a physical entity (like raw labor). The reason is that meta-physical entities can grow indefinitely while the growth of physical entities is bounded. The fallacy of imagining long-term economic growth driven by population growth was vividly illustrated by Blaug (1997, p. 68): [I]f the human race had sprung from a couple living in 10,000 BC and had grown since then, not at maximum biological rate but only at a modest 1 per cent per annum, the earth would now be a sphere of flesh several thousand light-years in diameter with a surface advancing into space at a rate many times faster than the rate at which light travels.

Against this background it is reassuring that, in retrospect, modern economic growth has been negatively associated with population growth and fertility rates (e.g. Brander and Dowrik, 1994; Ahituv, 2001; Li and Zhang, 2007; Herzer et al., 2012; Chatterjee and Vogl, 2018) and positively with growth of human capital (e.g. Krueger and Lindahl, 2001; Glaeser et al., 2004; Tamura et al., 2019). The association between education and growth becomes stronger in studies that replace schooling input (years of education) with schooling output (test scores) as a more precise measure of human capital (Hanushek and Woessman, 2008, 2012). With respect to productivity growth, Strulik et al. (2013) show for a panel of countries from 1950–2000 that growth of total factor productivity (TFP) is negatively associated with population growth and positively with measures of education. Also for a reduced sample of the G-7 countries, which accounted for more than 80 percent of the worldwide R&D spending during the observation period (Keller, 2009), the association between population growth and productivity growth is negative, suggesting that innovation or ideas-based growth may not be driven by an expansion of the raw labor force.

The negative association between fertility and growth of income and productivity is explained by unified growth theory as an interaction of technological progress and a child quantity-quality tradeoff at the family level (Galor and Weil, 2000; Galor and Moav, 2002; Galor, 2011; Dalgaard and Strulik, 2013; Madsen and Strulik, 2023). Technological progress increases the payoff of education and induces families to prefer less children and to invest more into the human capital of their offspring. Higher endowments with human capital increase the productivity and hence income of the next generation of adults, driving the virtuous circle of falling fertility and increasing human capital and per capita income.

An interesting and so far unexplored question is whether the virtuous circle of falling fertility and rising income per capita breaks down when fertility falls below the replacement level. To address this question, I refine the Jones (2022) model by an education decision. As an optimal response to rising incomes, households generate a child quantity-quality substitution that increases investments in human capital per capita of the next generation. In the long-run, education effort (measured by the shares of time and income spent on education) stabilizes at a constant level and fertility stabilizes below the replacement level. I show plausible conditions for which aggregate human capital grows without bound despite declining population. I calibrate the model with U.S. data and show that these optimistic long-run predictions can be generated by a model economy that replicates the historical trends of fertility, education, and income per capita that were observed in 20th century.

A couple of studies have addressed the question whether human capital accumulation can offset the effects of declining populations growth in models of ideas-driven growth (Dalgaard and Kreiner, 2001; Strulik, 2005; Strulik et al., 2013). This literature has not examined the case of negative population growth and has assumed that human capital is produced with constant returns on educational efforts. However, it has been argued that the "knife-edge" assumption of constant returns on human capital in the production of human capital is crucial for human capital to compensate for declining population growth and the possibility of long-run economic growth (Jones, 2022). For constant education, human capital-driven growth might even require increasing returns to education, which are not observed empirically (Jones, 2022). It is therefore important to expand the literature to include a discussion of the long-run consequences of *negative* population growth in the context of *declining returns* to investments in education. This is the purpose of the present work.

The paper is organized as follows. In the next section, I set up and discuss a basic model that explains a constant time investment in education and a constant level of fertility below the replacement rate. The model is used to prove and explain the main mechanics of economic growth with declining population. Then I extend the model so that it can explain the demographic transition and the rise in education in the 20th century. I calibrate these transitional dynamics with U.S. data. I use the model to show how adjustment dynamics lead towards constant education effort, constant fertility below replacement level, and high economic growth. I also consider an alternative specification of the ideas production function for which I can show the asymptotic stability of the steady state. The final section concludes the paper.

2. The Model

2.1. **Setup of Society.** We consider a model society of overlapping generations (OLG) of homogenous adults and their children. All decisions are made by adults and, by convention in the literature, we assume asexual reproduction, so when each adult has one child, the population is stationary. The current population of adults makes costly investments in the education of their children. Education builds human capital and enhances the productivity of children when they become workers in the next period. The adult time not used for raising children and education is made available as labor, which generates an income depending on the endowment with human capital. As in Jones (2022), physical capital and a savings motive are ignored, which in the current context means that there is no retirement period. Capital accumulation and a retirement period could be added to the model without the provision of new insights.

2.2. **Production and Aggregate Dynamics.** The production side of the economy is taken from Jones (2022) with one refinement: raw labor is replaced by human capital. Following the convention in the literature, aggregate human capital H_t is conceptualized as human capital per worker h_t times the number of workers L_t times the labor supply per worker ℓ_t, as stated in equation (1). Ignoring premature deaths, the next period's size of the workforce equals this period's size of the workforce times the fertility rate, as stated in equation (2). Output is a linear function of aggregate input and an iso-elastic function of the stock of ideas (knowledge), in which the degree of increasing returns is parameterized by σ, as in (3).[1]

[1] Parameters are denoted by Greek letters and are assumed to be non-negative.

$$H_t = h_t \ell_t L_t \tag{1}$$

$$L_{t+1} = n_t L_t \tag{2}$$

$$Y_t = A_t^{\sigma} H_t \tag{3}$$

$$A_{t+1} - A_t = A_t^{1-\beta} H_t^{\lambda}. \tag{4}$$

As in Jones (2022), we assume for simplification that there are no opportunity costs in the development of new ideas, which – in the sense of learning-by-doing – are an increasing function of aggregate input in production, as in (4).[2] The parameter λ defines the input elasticity of ideas (also known as the stepping-on-toes effect) and the parameter β specifies the degree at which new ideas are harder to find (whereby $1 - \beta$ is known as the standing-on-shoulders, effect). I mainly focus on Jones' (2022) specification of the ideas production function but will later also consider a more conservative way of generating ideas that eliminates the potential issue of explosive growth.

2.3. **Fertility and Education.** The adults of period t experience utility u_t from consumption c_t, the number of their children n_t, and the human capital of their children h_{t+1}. The latter assumption follows unified growth theory by modeling intergenerational altruism as parents' concern for their children's future income (Galor and Weil, 2000; Galor and Moav, 2002). Imposing a logarithmic form, the utility function is stated in equation (5).

Adults earn a wage w_t per unit of human capital h_t that is supplied as wage work. They have a time endowment of 1 unit and rearing a child requires ϕ units of time. Adults decide about the time s_t that they invest in education of their children and how to spend their disposable income on consumption c_t and education q_t. They thus face the time constraint $\ell_t = 1 - \phi n_t - s_t n_t$ and the budget constraint stated in equation (6).

$$u_t = \log c_t + \alpha \log n_t + \gamma \log h_{t+1} \tag{5}$$

$$w_t h_t (1 - \phi n_t - s_t n_t) = c_t + q_t n_t \tag{6}$$

[2] The model could be extended such that ideas production is a costly market activity, see Strulik et al. (2013). Since here the focus is on the general feasibility of long-run growth, I apply Occam's razor and strictly follow Jones' (2022) setup of the ideas production.

$$h_{t+1} = \nu q_t^{\theta} (s_t)^{\eta}, \qquad 0 < \theta, \eta < 1. \tag{7}$$

Human capital of the next generation is a positive function of the investments of resources and time of the current generation. For the basic model, I follow the standard assumption of unified growth theory that the adult (teacher) generation bears the time costs of education. A unit of teacher time corresponds to κ units of student time s_t^Y, where κ can be imagined as class size. In the Appendix, I show that this model is isomorphic to a more complex model where part of education (e.g. primary and secondary education) is determined by the parent generation and part of education (e.g. tertiary education) is determined by the child generation, taking into account foregone labor income in youth. We focus on the interesting case of diminishing returns on both types of investment such that the production function for human capital reads as stated in equation (7) with $\theta < 1$ and $\eta < 1$.

Households maximize utility (5) subject to the budget constraint (6) and human capital production (7). The solution of this problem (derived in the Appendix) is given by $c_t = w_t h_t/(1+\alpha)$ and

$$n_t = \bar{n} \equiv \frac{\alpha - \gamma(\eta + \theta)}{(1+\alpha)\phi} \tag{8}$$

$$s_t = \bar{s} \equiv \frac{\eta\gamma\phi}{\alpha - \gamma(\eta + \theta)} \tag{9}$$

$$q_t = \bar{q} w_t h_t \equiv \frac{\gamma\phi\theta}{\alpha - \gamma(\eta + \theta)} w_t h_t. \tag{10}$$

Due to the simple log-linear structure of the problem, households prefer a constant number of children $\bar{n}$, a constant time investment in education $\bar{s}$, and a constant expenditure share of education $\bar{q}$. In order to ensure an interior solution, I assume a sufficiently large weight of children in utility, $\alpha > \gamma(\eta + \theta)$. The comparative statics of the solution provide the child quantity-quality trade-off: a larger weight on human capital (γ) or a higher return to education (η or θ) induces more investment in education and lower fertility.

2.4. **Equilibrium and Steady State.** Following Jones (2022), I assume that markets are perfectly competitive such that workers earn a wage $w_t = A_t^{\sigma}$ per unit of human capital. Inserting this information and (9) and (10) into (7), the law of motion for human capital is obtained as

$$h_{t+1} = \nu \bar{q}^{\theta} A_t^{\sigma\theta} h_t^{\theta} \bar{s}^{\eta}. \tag{11}$$

Note that, in reduced-form, human capital of the next generation h_{t+1} is produced with *diminishing returns* to the current generation's human capital h_t because θ is less than one.

For optimal time investment in education, labor supply per worker is $\ell_t = 1 - \phi\bar{n} - \bar{s}\bar{n}$ and aggregate labor supply is $(1 - \phi\bar{n} - \bar{s}\bar{n})L_t$ such that the labor market equilibrium condition (1) can be written as

$$H_t = (1 - \bar{s}\bar{n} - \phi\bar{n})h_t L_t. \tag{12}$$

In order to facilitate the steady state analysis, I denote the growth rate of a variable x at time t by $g_{x,t} \equiv (x_{t+1}/x_t) - 1$ and the growth rate at the steady state by g_x. At a steady state, the ratio of the growth rates at time $t+1$ and time t equals 1. The evaluation of the equation (4) at the steady state provides:

$$\frac{g_{A,t+1}}{g_{A,t}} = \left(\frac{A_{t+1}}{A_t}\right)^{-\beta} \left(\frac{H_{t+1}}{H_t}\right)^{\lambda} \overset{!}{=} 1 \qquad \Rightarrow \qquad (1+g_A)^{\beta} = (1+g_H)^{\lambda} = (1+g_h)^{\lambda}\bar{n}^{\lambda}, \tag{13}$$

where I used the fact that $H_{t+1}/H_t = (h_{t+1}/h_t)(L_{t+1}/L_t)$ from (12) and $L_{t+1}/L_t = \bar{n}$ from (2) and (8). The evaluation of the equation (11) at the steady state provides:

$$\frac{1+g_{h,t+1}}{1+g_{h,t}} = \left(\frac{A_{t+1}}{A_t}\right)^{\sigma\theta} \left(\frac{h_{t+1}}{h_t}\right)^{\theta-1} \overset{!}{=} 1 \qquad \Rightarrow \qquad (1+g_A) = (1+g_h)^{(1-\theta)/(\sigma\theta)}. \tag{14}$$

Combining (13) and (14) provides the steady-state growth rate of human capital per capita:

$$1 + g_h = \left(\frac{1}{\bar{n}}\right)^{\frac{\lambda\sigma\theta}{\lambda\sigma\theta - \beta(1-\theta)}}. \tag{15}$$

Proposition 1. *For a declining population and $\lambda\sigma\theta > \beta(1-\theta)$, there exists a steady state with positive long-run growth of ideas ($g_A > 0$) and income per capita ($g_y > 0$).*

Proof. The population is declining for $0 < \bar{n} < 1$. The right hand side of (15) is thus larger than 1 if the exponent is larger than zero. This is the case for $\lambda\sigma\theta > \beta(1-\theta)$. Then, (15) implies that, at the steady state, negative population growth is associated with a positive growth rate of human capital per capita g_h. From (14) then follows a positive growth rate of ideas g_A. Income per capita is obtained from (1) and (9) as $Y_t/L_t \equiv y_t = A_t^{\sigma}(1 - \phi - \bar{s})h_t$. At the steady state, income grows thus at the positive rate $g_y = \sigma g_A + g_h$ since both g_A and g_h are positive. □

Corollary 1. *A steady state of positive growth of ideas and income does not require the assumption of constant marginal returns to investment in education.*

Proof. The condition in Proposition 1 is independent from the return to time investment in education (η). According to Proposition 1, positive growth is obtained in case of declining returns to investment of income in education ($\theta < 1$) iff $\theta/(1-\theta) > \beta/(\lambda\sigma)$. The condition has a non-empty solution set for feasible combinations of β, λ and σ. □

The corollary is important because it has been argued that human capital accumulation is an unconvincing solution to the problem of population decline since steady-state growth would require constant returns to education (see e.g. Jones, 2022). The corollary shows that this is not the case. For example, for the parameters assumed in the calibration by Jones (2022), which are $\beta = 2$, $\lambda = 3/4$, and $\sigma = 1$, the values of $\bar{n} = 0.86$ and $\theta = 0.795$ imply a growth rate of income per capita of 0.83 per generation. Assuming that a generation takes 30 years, these values mean that a population declining at a rate of 0.5 percent per year (the value assumed in Jones, 2022) generates a growth rate of income per capita of 2 percent per year.

The intuition for the result is the following. If the condition of Proposition 1 is met, ideas, and hence income and education expenditure, will grow fast enough to offset the diminishing returns on education spending, allowing per capita human capital to grow fast enough to offset the negative impact of population decline on total human capital growth. In terms of equation (11), we see that human capital can steadily grow despite diminishing returns with respect to time input (s_t) and human capital input (h_t) because the level of A_t is growing.

For a given input of time and human capital, later born generations accumulate more human capital because the knowledge base is larger than for previous generations. In other words, for given effort (study time and income share of education), the market value of the learned skills increases because there is more knowledge to learn. Notice that this feature does *not* imply that later born generations experience a higher (Mincerian) return to education. To see this, recall that labor income is given by $w_t h_t$, in which w_t is the wage per unit of human capital. The Mincerian return to education is thus given by $\rho_t \equiv \partial(\log h_{t+1})/\partial \tilde{s}^Y_{t+t}$, in which s^Y_{t+1} is the time that the young generation spends on education (Mincer, 1974). Suppose that the time that the young (student) generation spends on education is proportional to the time that the old generation spends on educating the young, $\tilde{s}^y_{t+1} = \kappa s_t$, in which κ denotes the factor of proportionality.

Proposition 2. *The return to education is given by $\rho = \eta/\bar{s}$. It is constant over time since the time spent on education is constant over time.*

Proof. Insert the proportionality assumption into (7), apply the definition of the Mincerian return to education, and insert (9). □

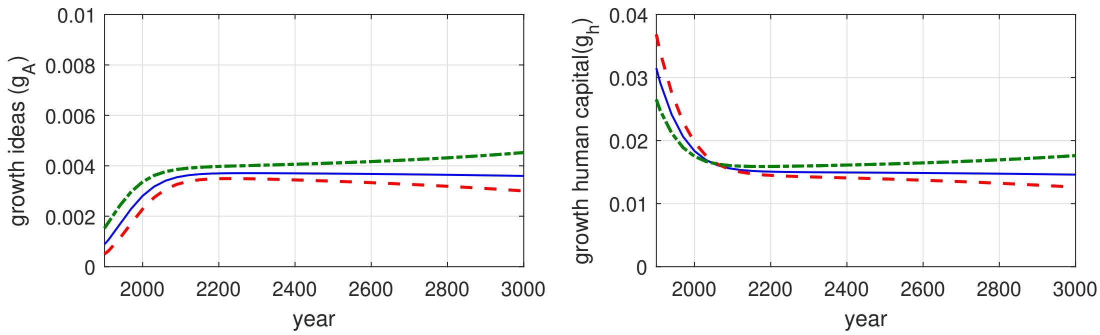
Since the return to education is constant over time, younger cohorts do not experience a higher return to education compared to older cohorts. The result from Proposition 2 is thus consistent with Mincerian wage regressions that do not reveal such a pattern. In the extended model (discussed below), however, there are adjustment dynamics and the return to education is only constant at the steady state where s_t is constant. While this prediction appears to be empirically plausible, it is not needed for the main result from Proposition 1. This is shown in the Appendix using a model variant in which the return to education is by design constant on and off the steady and therefore independent of the time spent on education.

2.5. **Stability.** A potential limitation of the model described above is that the steady state is not asymptotically stable. The reason is the iso-elastic function of ideas growth in conjunction with the overlapping generations structure. This instability is, in principle, also inherent in infinite-horizon, representative agent models. There, however, initial education is chosen such that the economy moves along a saddlepath towards the steady state. The appropriate choice of the initial values is enforced by the transversality condition that applies at the end of the infinitely long planning horizon of the representative agent (for a detailed exposition in the context of the neoclassical growth model, see Barro and Sala-i-Martin (2004, Chapter 2.5, 2.6).

In overlapping generations (OLG) models, the initial condition for human capital is not a choice variable and there is no transversality condition at the end of the planning horizon. Intuitively, initial human capital is thought of as being inherited from the last generation (which did not take the development of the dynasty in the infinite future into account). In this sense, the economy moves along the steady state only by coincidence. It can be shown, however, that for plausible sets of parameters and initial values, divergence from the steady state is slow, i.e. the economy behaves for a long time as if at a steady-state.

An example of adjustment dynamics is shown in Figure 1. A generation is assumed to take 30 years and growth rates are expressed in annual rates. The parameters and initial conditions in the year 1700 are shown below the Figure. A detailed calibration of the model will be provided in Section 3 for an extended model with variable fertility and education. Here, we consider just an illustration. The parameters imply that $\bar{n} = 0.86$ such that the population declines at an annual rate of 0.5 percent (the long-run UN projection for population growth). Along the steady state, ideas grow at a annual rate of 0.38 percent and human capital grows at a rate of 1.52 percent, implying that income per capita grows at 1.9 percent annually (which corresponds to the U.S. average for the last century).[3]

FIGURE 1. Steady State and Adjustment Dynamics



Blue (solid) $h_0 = 1.0$; red (dashed) lines: $h_0 = 0.1$, green (dash-dotted) lines: $h_0 = 10$. Parameters: $\sigma = 1$, $\lambda = 0.75$, $\beta = 2.0$, $\theta = 0.80$, $\alpha = 0.39$, $\gamma = 0.26$, $\eta = 0.07$, $\phi = 0.137$, $\nu = 1.79$. Initial values: $L_0 = 1$, $A_0 = 100$.

We consider three initial values for human capital per person, covering two orders of magnitude. The economy starting with initial human capital $h_0 = 1$ moves close to the steady state for a millennium without discernable deviation (blue solid lines). The economy starting with $h_0 = 0.1$ (red dashed lines) moves close to the steady state until around the year 2500 and then deviates visibly. The economy with initial human capital $h_0 = 10$ (green dash-dotted lines) starts to deviate visibly from the steady after about 2300. Until the year 2100, i.e. until the end of the period for which UN provides projections of population growth, the curves move visibly in sync akin confidence bands. Although the steady state is unstable with visible deviations over the next millennium, these deviations appear to be insignificant for the next centuries.

Adjustment dynamics with accelerating growth of ideas (like the green lines in Figure 1) also support the main argument of this paper, namely that positive long-run growth of ideas is possible with declining population growth. One could argue, however, that accelerating growth is implausible and that an asymptotically stable growth is a desirable outcome. In this regard it is interesting that the feature of accelerating growth of ideas can easily be avoided by assuming less favorable conditions for ideas generation. This is shown in the next section.

[3]The matlab code for these examples is available at `http://www.holger-strulik.org/papers.htm`.

2.6. **An Alternative Specification.** We consider the model as specified above but replace the iso-elastic ideas production function (4) with a logistic function:

$$A_{t+1} = (1 + g_{A,t})A_t, \qquad g_{A,t} \equiv \frac{\bar{g}_A}{1 + \omega \exp(-H_t/A_t)}. \tag{16}$$

The logistic growth function (16) inherits the main features from the isoelastic growth function, namely that the growth rate is declining in the stock of ideas A_t and increasing in the stock of human capital H_t. The crucial new feature is that ideas growth is bounded from above by $\bar{g}_A$. Notice that the logistic growth function preserves the crucial feature of the Jones (2022) model: without individual human capital accumulation and declining population, aggregate human capital H_t declines at the same rate as the population and the growth rate of ideas converges to its lower bound $\bar{g}_A/(1+\omega)$, which is close to zero for large values of ω.[4]

Proposition 3. *Consider the endogenous growth model (1)–(12), where logistic growth of ideas (16) replaces iso-elastic growth (4). Then there exists an asymptotically stable maximum long-run growth rate $\bar{g}_A$ if*

$$\bar{n} > (1 + \bar{g}_A)^{\frac{1-(1+\sigma)\theta}{1-\theta}}. \tag{17}$$

For the proof, notice from (16) that the economy converges to maximum growth when aggregate human capital H_t grows at a higher rate than ideas A_t, i.e. for $(1+g_h)\bar{n} > 1+g_A$. Since the alternative specification preserves the fertility and education part of the model, condition (14) also applies at the steady state of the alternative model. Using it to replace g_h in the maximum growth condition provides condition (17).

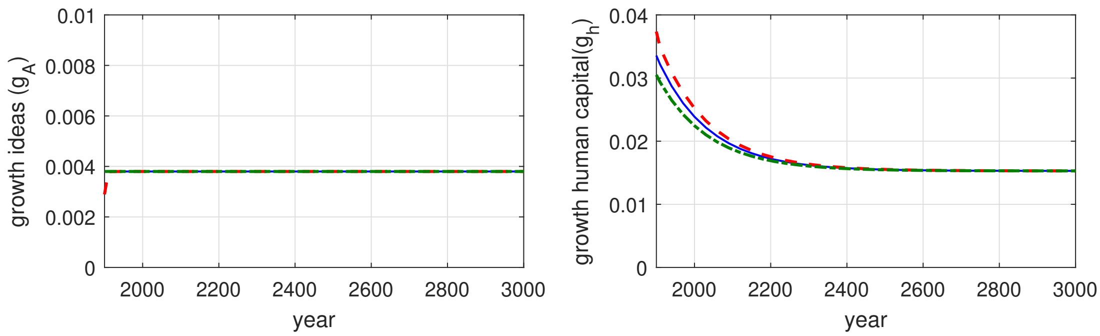
Since $0 < \theta < 1$, the steady state with declining population growth ($\bar{n} < 1$) and economic growth at its maximum $\bar{g}_A$ exists for $(1 + \sigma)\theta > 1$. Taking $\sigma = 1$ from Jones (2022), the condition simplifies to $\theta > 1/2$. Suppose $\theta = 0.7$ and $\sigma = 1$ and that the length of a generation is 30 years. If the maximum annual growth of ideas is 0.5 percent, then asymptotic stability requires $\bar{n} > 0.8191$, i.e. the population must not decline by more than 0.66 percent annually for asymptotically stable maximum growth. If $\bar{g}_A$ is 1.5 percent annually, the population can decline by up to 1.96 percent per year at the asymptotically stable steady state.

Figure 2 shows the resulting adjustment dynamics for the example economy introduced with Figure 1. All parameters and initial values are as given below Figure 1 and $\omega = 100$. For

[4] Convergence to exactly zero could be achieved by implementing a somewhat more complicated logistic function.

all considered initial values, the economy converges quickly to the steady state of maximum growth. In conclusion, an asymptotically stable state of high economic growth is achievable with a continuously declining population.

FIGURE 2. Alternative Specification: Steady State Convergence



Blue (solid) lines: $h_0 = 1.0$; red (dashed) lines: $h_0 = 0.1$, green (dash-dotted) lines: $h_0 = 2.0$. Parameters and initial values as for Figure 1 and $\omega = 100$.

3. U.S. Demographic and Economic Development 1900–2300

3.1. **Setup.** This section introduces a refinement of the model that generates adjustment dynamics for fertility and population growth. The main purpose of this section is to show that perpetual economic growth in a future of declining population is consistent with the observation that, in the past, economic growth was also driven by population growth. For this, we consider the smooth adjustment dynamics along the demographic transition, in which fertility drops from around 4 children per women below the replacement level.

Specifically, I assume that the marginal return on investing the first unit of income in education is not infinite. Human capital is thus produced as $h_{t+1} = \nu(q_t + \psi)^\theta s_t^\eta$. The human capital acquired without financial investment can be conceptualized as the intergenerational transmission of production knowledge for subsistence farming. With this refinement, the solution for fertility and education investments is obtained as (see Appendix for details):

$$n_t = \frac{[\alpha - \gamma(\eta + \theta)]\, w_t h_t}{(1+\alpha)\,[\phi w_t h_t - \psi]} \tag{18}$$

$$s_t = \frac{\eta\gamma(\phi w_t h_t - \psi)}{[\alpha - \gamma(\eta + \theta)]\, w_t h_t} \tag{19}$$

$$q_t = \frac{\gamma\theta\phi w_t h_t - (\alpha - \eta\gamma)\psi}{\alpha - \gamma(\eta + \theta)}. \tag{20}$$

Notice that the solution converges to the solution of the basic model as income grows without bound; i.e. $n_t = \bar{n}$, $s_t = \bar{s}$, and $q_t = \bar{q} w_t h_t$ for $w_t h_t \to \infty$. The model differs from the basic model only in terms of transitional dynamics for fertility and education. Starting from low income and productivity, fertility adjusts from above and investments in education adjust from below. There exists a range of parameters and initial values for which households do not invest in education but for brevity we focus on the interior solution. The exposition first considers the basic model (with iso-elastic ideas growth) and then the alternative version (with logistic ideas growth).

3.2. **Calibration.** To better align the numerically specified model with real data, I assume that a generation takes 30 years and convert the model outcomes into annual values. For example, a growth rate of g_x per generation translates into a growth rate of $(1 + g_x)^{1/30} - 1$ per year. In order to save notation, I use the symbol g_x also for the annual growth rate. The calibration period begins in the year 1900 at the dawn of the U.S. fertility transition (Reher, 2004). Fertility is gradually declining, with the population growth rate approaching a lower bound at $\bar{g}_L = \bar{n}^{1/30} - 1$. During the 20th century the U.S. did not only experience the fertility transition but also an unprecedented surge in investment in education. This expansion needs to be conceptualized as a temporary phenomenon since both the share of lifetime devoted to education and the share of income devoted to education are bounded from above.

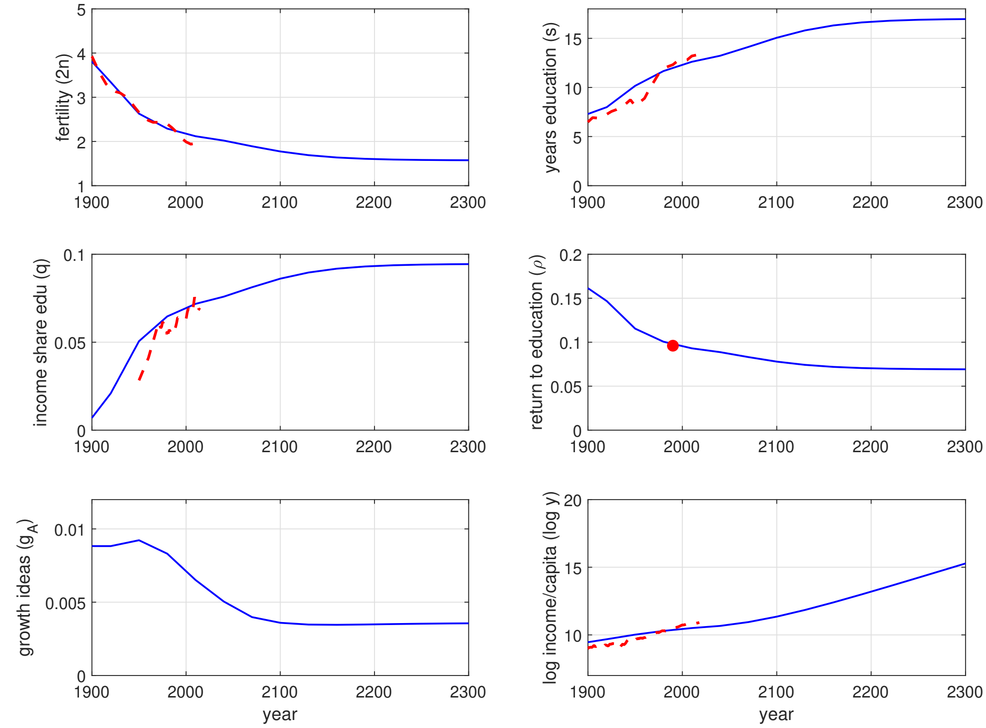
The targeted historical paths of the total fertility rate, years of education, the education expenditure share in GDP, and GDP per capita are shown by red (dashed) lines in Figure 3. The data for fertility are from from UN (2022). The time path shown in the upper-left panel of Figure 3 depicts the five-year moving average in order to smooth out the baby boom of the 1950s and 1960s, which cannot be explained by the model. The data for years of education of the population aged 25+ are from Lee and Lee (2016) and the data for GDP per capita are from the Maddison Project Database 2020 release (Bolt and van Zanden, 2020). The education expenditure data are from Snyder et al. (2016). The center-left panel of Figure 3 shows the GDP share of total education expenditure (private and public) for primary, secondary, and tertiary education. The lion's share is public expenditure (which, for example, accounts for 80 percent of total expenditure in 2015, the last point of observation).

The technology parameters are calibrated as suggested in Jones (2022), i.e. constant returns of ideas in goods production ($\sigma = 1$), a stepping-on-toes effect of $\lambda = 3/4$, and, with $\beta = 2$, steeply decreasing returns of ideas in ideas production, as suggested in Bloom et al. (2020). The remaining parameters are jointly determined to fit the following calibration targets: (i) An annual decline of the population by 0.5 percent around the year 2100. This value matches the global fertility rate projected by UN (2022) for the year 2100 (the end of the UN projection horizon). It is also the value assumed in Jones (2022). (ii) A long-run growth rate of per capita income of 2.0 percent per year (slightly above the 20th century average). (iii) A GDP share of education expenditure of 6.8 percent, which matches the observed value in the last year of observation (2015). (iv) A time expenditure on education of 13.2 years in 2020, which matches the observed value in the last year of observation. In model time, this means that a young person spends $s^y = 13.2/60$ time units on education. The factor of proportionality between teacher time and student is set to $\kappa = 16.5$, reflecting an average class size of 16.5 students per teacher (according to NCES, 2022).[5] (v) A return to education of 9.8 percent in 1990, as reported in Psacharopoulos (1994), a study that has been frequently used for calibration exercises (e.g. Hall and Jones, 1999). (vi) Finally, I set the initial values $A(0) = L(0) = h(0) = 1$ for the initial year 1500 and adjust ν and ψ such that the fertility transition is initiated in the year 1900 and income growth approximates average income growth 1900-2020. The calibration provides the estimates $\alpha = 0.39$, $\gamma = 0.26$, $\eta = 0.07$, $\phi = 0.138$, $\theta = 0.83$, $\psi = 4.70$, and $\nu = 0.114$. The predicted, non-targeted value of the time cost of children (0.138) is close to the value of 0.125 assumed in Jones (2022). The calibration provides a steady-state fertility rate of 0.786. This implies that, at the steady state, the population declines at an annual rate of 0.8 percent, ideas grow at an annual rate of 0.38 percent, and income per capita grows at a rate of about 2 percent.

3.3. **Results.** The predicted time paths are shown by blue (solid) lines in Figure 3. In the upper left panel, fertility per person (n) is multiplied by two to be comparable to the TFR (fertility per woman). In the bottom right panel, predicted income is normalized such that it coincides with actual GDP per capita in 1980. The model thus targets only the slope but not the level of the path of GDP per capita. The model predicts a somewhat late fall in fertility below the replacement level but otherwise agrees fairly well with the data. During the transition period, growth of productivity and income are negatively correlated with fertility and positively with education investments and human capital. The center right panel shows the predicted return

[5] Following conventional unified growth theory (e.g. Galor, 2011), the simple model focusses on the opportunity cost of education of the teacher generation and ignores the opportunity cost of the student generation. See the Appendix for a more complex model that takes the opportunity cost of the young into account.

FIGURE 3. Demographic Transition and Growth with Declining Population: 1900 – 2300



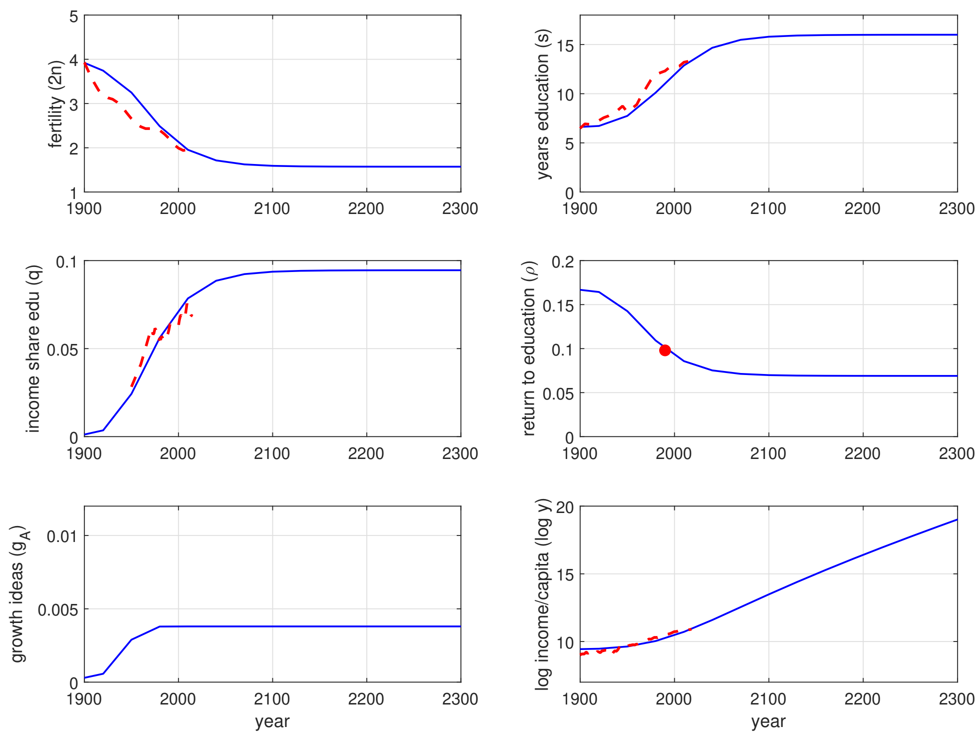
Blue (solid) lines: model prediction Red (dashed) lines and circles: data.

to education (per student year). It is declining along the adjustment path as years of education are increasing. Later born cohorts experience a lower return because they spend more years on education. The model predicts relatively high returns to education but in order of magnitude the slope of the curve agrees fairly well with conventionally estimated values. For example, based on Psacharopoulos (1994), Hall and Jones (1999) assume that the return to education is 13.4 percent for the 1 to 4 years of education, 10.1 percent for the 5-8 years of education, and 6.8 percent for 9 or more years of education.

For a correct interpretation of the results, notice that the model maintains the feature of conventional ideas-driven growth models that fertility and population growth contribute *positively* to the growth of ideas and income (see e.g. equation (13)). Economic growth could be higher if parents increased both education *and* fertility. This behavior, however, was not observed over the 20th century. The model shows that the actual behavior of investing less in the number of children and more on education is consistent with steady growth of ideas and income even if the number of children falls below replacement level.

3.4. **Results: Alternative Specification.** Finally, we inspect results for the alternative specification of idea growth (16). I set $\bar{g}_A = 0.12$, implying that the economy converges to the same steady state as the benchmark economy from Figure 3. I set $\omega = 100$, implying that an otherwise similar economy without growth of human capital (such as Jones, 2022) converges to a low steady state growth rate of ideas of 0.0038 percent, a value close to zero (one hundredth of $\bar{g}_A$). To fit the transitional dynamics in the data, I adjust $\nu = 11$ and $\psi = 0.08$. All other parameter values are kept from the previous calibration.

FIGURE 4. Alternative Specification: Steady Growth with Declining Population: 1900 – 2300



Blue (solid) lines: model prediction Red (dashed) lines and circles: data.

Figure 4 shows the implied transitional dynamics. Results look similar to Figure 3 but with different transitional dynamics for ideas growth. The conventional iso-elastic function of ideas growth used in Romer (1990)–Jones (1995)–style models of endogenous growth ($g_A = A^{-\beta} H^{\lambda}$) has the inconvenient feature that the growth rate of ideas is high when the stock of ideas is low. The logistic growth function replaces this feature with gradual growth of ideas, consistent with the historical observation of a gradual growth ideas measured in terms of patents (Madsen and Strulik, 2023).

4. Conclusion

Conventional ideas-driven growth models, in which ideas are generated by the input of physical labor suggest that a declining population will lead to economic stagnation. In this paper, I have extended an ideas-driven growth model by an education decision and human capital as input in the production of goods and ideas. I have shown that this refinement leads to a more optimistic outlook. Under plausible assumptions, the model predicts perpetual economic growth with declining population. Economic growth is micro-founded in that fertility and human capital growth are generated by optimal decisions at the household level but the condition for positive growth is not based on specific assumptions about household preferences. Long-term economic growth requires mild assumptions about the technologies used in the production of goods, ideas, and human capital. Long-term growth is predicted to be sustainable for an ideas production technology that would lead to stagnation in the absence of education and for a human capital technology with diminishing returns on investment in education. A model calibrated to replicate the trends in fertility, education, and income observed in the U.S. over the course of the 20th century predicts steady economic growth for declining population in the 21st and 22nd centuries.

Many scholars have argued that steady positive population growth on planet Earth would not be sustainable. It can be argued that steady negative population growth is also unsustainable due to indivisibilities in the organization of work and society. A reasonable interpretation of the results should therefore not take the steady state literally as a prediction for the infinite future. Given the projected peak of the world population of about 12 billion in 2050 (UN, 2022) and the calibrated rate of population decline of 0.5 percent annually, it would take 4600 years before just one human lives on earth. This would be the literal result of the ”empty planet”, and it seems reasonable that humanity would run into indivisibility problems much sooner. On the other hand, it would take about 270 years for the human population to decrease from 12 billion in 2050 to 3 billion in 2330. The world population was 3 billion in 1960 and we know from experience that indivisibility constraints were not a barrier to ideas-driven growth at that time. Therefore, an annual 0.5 percent decline in world population seems conceivable for a planning horizon up to the year 2300 (which extends the horizon of the UN population projections by 200 years). The results of this study suggest that such a population decline can be accompanied by sustained economic growth, as the smaller number of future generations is offset by the greater knowledge acquisition of later-born individuals.

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Appendix

4.1. **Solution of the Basic Household Problem.** The Lagrangian associated with the problem of maximizing (5) subject to (6) and (7) reads:

$$L = \log c_t + \alpha \log n_t + \gamma\theta \log q_t + \gamma\eta \log s_t + \mu\left[w_t h_t(1 - \phi n_t - s_t n_t) - c_t - q_t n_t\right]$$

The first order conditions are:

$$\frac{1}{c_t} = \mu \tag{A.1}$$

$$\frac{\alpha}{n_t} = \mu\left[w_t h_t(\phi + s_t) + q_t\right] \tag{A.2}$$

$$\frac{\gamma\theta}{q_t} = \mu n_t \tag{A.3}$$

$$\frac{\gamma\eta}{s_t} = \mu w_t h_t n_t. \tag{A.4}$$

From (A.1) and (A.2) we have $\alpha c_t = n_t w_t h_t(\phi + s_t) + q_t + n_t$. Inserting this into the budget constraint (6) provides

$$c_t = w_t h_t/(1 + \alpha). \tag{A.5}$$

Inserting (A.1) and (A.4) into (A.2) provides $\alpha = \mu w_t h_t n_t \phi + \gamma\theta + \gamma\eta$. Subsequently replacing μ_t from (A.1) and c_t from (A.5) and solving for n_t provides (A.8) in the main text. Inserting (A.1), c_t, and n_t into (A.3) provides q_t from (10) in the main text. Inserting (A.1), c_t, and n_t into (A.4) provides q_t from (9) in the main text.

4.2. **Globally Constant Returns to Education.** Suppose human capital is accumulated as

$$h_{t+1} = \nu q_t^{\theta} \cdot \mathrm{e}^{\bar{\rho} s_t},$$

which replaces (7). Maintaining the assumption of proportionality between the time spent on education by the student generation and the teacher generation, the implied Mincerian return to education is obtained as $\bar{\rho}$. It is independent from education effort. Proceeding as in Section 2, the solution of the household problem is obtained as:

$$n_t = \bar{n} \equiv \frac{\gamma\bar{\rho}}{(1 + \alpha)}$$

$$s_t = \bar{s} \equiv \frac{\alpha - \gamma(\phi\bar{\rho} + \theta)}{\gamma\bar{\rho}}$$

$$q_t = \bar{q} w_t h_t \equiv \frac{\theta}{\bar{\rho}} w_t h_t.$$

Inserting s_t and q_t into h_{t+1} provides $h_{t+1} = \nu\bar{q}e^{\bar{\rho}\bar{s}} A_t^{\sigma\theta} h_{\theta-1}$, which is structurally identical to (11), implying the same results as for the model from Section 2.

4.3. **Solution of the Extended Household Problem.** The Lagrangian associated with the extended problem reads:

$$L = \log(c_t) + \alpha \log n_t + \gamma\theta \log(q_t + \psi) + \gamma\eta \log(s_t) + \mu\left[w_t h_t(1 - \phi n_t - s_t n_t) - c_t - q_t n_t\right]$$

The first order conditions are:

$$\frac{1}{c_t+} = \mu \tag{A.6}$$

$$\frac{\alpha}{n_t} = \mu\left[w_t h_t(\phi + s_t) + q_t\right] \tag{A.7}$$

$$\frac{\gamma\theta}{q_t + \psi} = \mu n_t \tag{A.8}$$

$$\frac{\gamma\eta}{s_t} = \mu w_t h_t n_t. \tag{A.9}$$

Proceeding as described for the basic problem provides the solution (18)–(20) of the main text.

4.4. **Some Education Time is Set by the Student Generation.** In this extension, I generalize the model by assuming that only part of the child generation's study time (e.g. primary and secondary education) is controlled by the parent generation and addressed as compulsory schooling. Another part (e.g. tertiary education) is controlled by the child generation. I maintain the assumption of a constant relation between student and teacher time (constant class size). The time that the child generation of period t spends on education is identified by superscript y, which means that the young generation spends $s_t^y = \kappa s_t$ units on compulsory schooling and $\tilde{s}_t^y = \kappa \tilde{s}_t$ units of voluntary schooling. In order to simplify the problem we assume that the human capital production function is multiplicatively separable in compulsory and voluntary education. Measured in teacher time, it reads $h_{t+1} = \nu(q_t + \psi)^\theta s_t^\eta \tilde{s}_t^\omega$. The budget constraint of parents is modified to $w_t h_t(1 - \phi n_t - s_t n_t - \tilde{s}_t n_t) = c_t + q_t n_t$, which replaces (7). The rest of the parent's problem is as specified for the extended problem in the main text. Proceeding as before and noting that $\tilde{s}_t$ is set by the child generation, we obtain the optimal solution:

$$n_t = \frac{\left[\alpha - \gamma(\eta + \theta)\right](\bar{c} + w_t h_t)}{(1 + \alpha)\left[(\phi + \tilde{s}_t)w_t h_t - \psi\right]} \tag{A.10}$$

$$s_t = \frac{\left[(\eta\gamma(\phi + \tilde{s}_t)\right] w_t h_t - \eta\gamma\psi}{\left[\alpha - \gamma(\eta + \theta)\right] w_t h_t} \tag{A.11}$$

$$q_t = \frac{\gamma\theta(\phi + \tilde{s}_t)w_t h_t - (\alpha - \eta\gamma)\psi}{\alpha - \gamma(\eta + \theta)}. \tag{A.12}$$

The solution is isomorph to the solution (18)–(20) of the main text. The opportunity cost of voluntary education of the young generation is lost income from work. Thus, in contrast to the model from the main text, the young generation is economically active. We assume that children do not work during compulsory education. The young generation maximizes utility from consumption which is equivalent to maximize income, which is given by

$$(1 - \kappa\tilde{s}_t)\nu(q_t + \psi)^\theta s_t^\eta \tilde{s}_t^\omega.$$

The first order condition with respect to $\tilde{s}_t$ provides the solution $\tilde{s}_t = \omega/[\kappa(1 + \omega)]$ and thus $\tilde{s}_t^y = \bar{s}^y \equiv \omega/(1 + \omega)$. Aggregate supply of human capital in period t consists of human capital supplied by the parent and child generation. Assuming that children spend a fraction δ of the childhood period in either tertiary education or wage work and noting that a parent has n_t

children, aggregate human capital is obtained as

$$H_t = [1 - (s_t + \bar{s}^y + \phi)n_t)L_t + (\delta - \kappa\bar{s}^y)n_t L_t.$$

Since n_t and s_t converge to constants for rising income, the long-run solution of the model is isomorphic to the solution in the main text.