Abstract. This paper proposes a theory for the social evolution of obesity. It considers a society in which individuals experience utility from consumption of food and non-food, the state of their health, and the evaluation of their appearance by others. The theory explains under which conditions poor persons are more prone to be overweight although eating is expensive and it shows how obesity occurs as a social phenomenon such that body mass continues to rise long after the initial cause (e.g. a lower price of food) is gone. The paper investigates the determinants of a steady state at which the median person is overweight and how an originally lean society arrives at such a steady state. Extensions of the theory towards dietary choice and the possibility to exercise in order to lose weight demonstrate robustness of the basic mechanism and provide further interesting results.

Keywords: Obesity Epidemic, Social Dynamics, Social Multiplier, Income Gradient, Feeling Fat, Feeling Unhealthy, Fat Tax.

1. Introduction

Since about the last quarter of the 20th century we witness an unprecedented change in the phenotype of human beings. In the US, for example, the share of overweight (obese) persons was almost constant at about 45 percent (15 percent) of the population in the years 1960 to 1980. Since then, the share of overweight adults rose to 64.7 percent in the year 2008 and the share of obese adults rose to 34.3 percent (Ogden and Carroll, 2010). If these trends continue, by 2030, 86 percent are predicted to be overweight and 51 percent to be obese (Wang et al., 2008). The phenomenon of increasing waistlines is particularly prevalent in the US but is also observed globally (OECD, 2010, WHO, 2011). The world is getting fat (Popkin, 2009).

Obesity entails substantial health costs. Obese persons are more likely to suffer from diabetes, cardiovascular disease, hypertension, stroke, various types of cancer and many other diseases (Field et al., 2001, Flegal et al., 2005). As a consequence, obese persons spend not only more time and money on health care (Finkelstein et al., 2005, OECD, 2010) but they also pass away earlier. For example, compared to their lean counterparts, 20 year old US Americans can expect to die about four years earlier when their BMI exceeds 35 and about 13 years earlier when their BMI exceeds 45 (Fontaine et al., 2003). According to one study, obese persons actually incur lower health care costs over their life time due to their early death (van Baal et al., 2008).

The simple answer for why people are overweight is that they like to eat more than their body can burn. In the US, for example, 70 percent of the adult population in the year 2000 said that they eat “pretty much whatever they want” (USDA, 2001). Although a fully satisfying answer is certainly more complex, involving biological and psychological mechanisms, perhaps the most striking observation in this context is that overeating seems not to be driven by affluence. At the beginning of the 20th century, when the developed countries were certainly no longer constrained by subsistence income, the English physiologist W.M. Bayliss wrote

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1Overweight is defined as a body mass index (BMI) above 25 and obesity as a BMI above 30. In this paper we thus apply the inclusive definition of overweight by the WHO (2011), according to which obese persons are also regarded as overweight. Some other studies apply an exclusive definition according to which only persons with BMI between 25 and 30 are regarded as overweight. The BMI is defined as weight in kilogram divided by the square of height in meters.

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that “it may be taken for granted that every one is sincerely desirous of avoiding unnecessary consumption of food” (Bayliss, p. 1). Indeed, caloric intake per person in the US remained roughly constant between 1910 and 1985. But it then rose by 20% between 1985 and 2000 (Putnum et al., 2002, see also Cutler et al., 2003 and Bleich et al., 2008).

Across the population, within countries, the historical association between affluence and body mass actually changed its sign over the 20th century; “where once the rich were fat and the poor were thin, in developed countries these patterns are now reversed.” (Pickett et al., 2005). But while it is true that the severity of overweight and obesity is much stronger for the poor than for the non-poor (Joliffe, 2011), it is also true that persons from all social strata are equally likely to be overweight (in the US) and that the secular increase of overeating and overweight is equally observed among – presumambly richer – college graduates and non-college educated persons (Ruhm, 2010). Across countries, obesity and calorie consumption appear to be more prevalent in unequal societies (Pickett et al., 2005).

The evolving new human phenotype cannot be explained by genetics because it occurred too rapidly (e.g. Philipson and Posner, 2008). It has to be conceptualized as a social phenomenon. With affluence being an unlikely candidate, the question arises what has caused the social evolution of overweight and obesity? The most popular factors suggested in the literature are decreasing food prices, decreasing effective food prices through readily available convenience foods and restaurant supply, and less physical activity on the job and in the household (see e.g. Finkelstein et al., 2005, OECD, 2010). But these explanations entail some unresolved puzzles with respect to the timing of the obesity epidemic.

The most drastic changes of potential causes of obesity occurred well before obesity prevalence became a mass phenomenon. The price of food declined substantially from the early 1970s through the mid 1980s but changed little thereafter, when the obesity epidemic took off. Eating time declined substantially from the late 1960s to the early 1990s, but stabilized thereafter (see Ruhm, 2010). Likewise, the gradual decline in manual labor and the rise of

\[\text{Many, but not all, empirical studies of the income obesity nexus find it unambiguously negative for all subgroups of society. For example, Lakdawalla and Philipson (2009) document a hump-shaped association of BMI and income for male US American workers but a monotonously negative association for female workers.}\]
labor saving technologies at home began before the rapid rise in obesity and slowed down afterwards (Finkelstein et al., 2005). This means that calories expended have not decreased much further since the 1980s (Cutler et al., 2003).

From these facts some studies conclude that food prices and caloric expenditure are unlikely to be major contributors to the evolution of obesity because the prevalence of obesity continues to rise after the alleged causes have (almost) disappeared. The present paper proposes an alternative conclusion based on social dynamics. It explicitly considers that one’s appearance is evaluated by others. The social disapproval for displaying an overweight body is continuously but slowly updated by the actual observation of the prevalence of overweight in society. This view provides (i) a social multiplier that amplifies the “impact effect” of exogenous shocks, and (ii) an explanation for why we observe an evolving human phenotype long after the impact effect is gone.

The theory establishes two exclusively existing, stable, and qualitatively distinct social equilibria. At one equilibrium the median person is lean and after an exogenous shock that favors overeating (e.g. lower food prices) social pressure leads society back to the lean equilibrium. This means that, although there are overweight and obese persons in society, obesity is not an evolving social problem. At the other equilibrium the median is overweight and after an exogenous shock that favors overeating, society at large converges towards an equilibrium where people are, on average, heavier than before. The historical evolution of BMI in the US., for example, is conceptualized according to the theory as a stable lean steady state until the 1970s and a transition towards a stable obese steady state afterwards.

The theory explicitly takes into account that preferences and income vary across individuals. Holding income constant it predicts that people with a high preference for food consumption are heavier. Holding preferences constant it predicts that poorer people are heavier, at least if income is sufficiently large and the elasticity of substitution between food and non-food is larger than unity. The reason is that rich persons inevitably consume more (food or non-food) than poor ones. Given non-separable utility, they thus experiences higher marginal utility from being lean (or less overweight) and consequently they consume fewer calories. A poor
person, in contrast, puts less emphasis on the evaluation of her appearance by others and on the health consequences of being overweight because the scale of consumption (food or non-food) is low. Due to the lower emphasis on weight a larger share of experienced utility results from food consumption, in particular if food prices are low compared to other goods. Since the median is poorer in unequal societies, the theory predicts, that, ceteris paribus, unequal societies are more afflicted by the obesity epidemic.

In Section 3 it is shown that the social multiplier produces some perhaps unexpected non-linearities. In particular, an obesity related health innovation (e.g. beta-blockers, dialysis) can go awry. The impact effect of such an innovation is initially better health for everybody. But the lower health consequences of being overweight induces some people to eat more and put on more weight. This may set in motion a bandwagon effect and convergence towards a new steady state at which society is, on average, not only heavier but also less healthy than before the health innovation.

The basic model fails to capture some further aspects of the obesity epidemic, most importantly the role of energy-density of food and that of physical exercise. Section 4 thus extends the model to account for these factors and shows that all basic results are preserved under mild conditions. It also derives some refinements of the original theory. For example, while richer people, continue to be predicted to be, ceteris paribus, less overweight, leaner bodies are no longer necessarily a consequence of eating less. Instead, richer people are predicted to exercise more for weight loss. In a two-diet model, a rising energy density of the less healthy diet is predicted to increase body mass if the diet is sufficiently cheap and its consumer sufficiently poor. If this applies to the median person, society at large is predicted to get heavier due to the social multiplier.

There exists some evidence supporting the basic assumption that being overweight generates less disutility if many others are overweight or obese as well, that is if the prevalence of being overweight in society is high. Blanchflower et al. (2009) find that females across countries are less dissatisfied with their actual weight when it is relatively low compared to average weight. Using the German Socioeconomic Panel they furthermore find that males, controlling for their
actual weight, experience higher life satisfaction when their relative weight is lower. In the US, about half of the respondents to the Pew Review (2006) who are classified as overweight according to the official definition characterize their own weight as “just about right”. Etilé (2007) provides similar results for France and argues that social norms and habitual BMI affect ideal BMI, which in turn influences actual BMI. Christakis and Fowler (2007) show how obesity spreads from person to person in a large social network and find that a person’s chances to become overweight increase by 57 percent if he or she has a friend who became obese. Trogdon et al. (2008) find that for US adolescents in 1994-5 individual BMI was correlated with mean peer BMI and that the probability of being overweight was correlated with the proportion of overweight peers. Comparing different periods of observation from the National Health and Nutrition Examination Survey, Burke et al. (2010) show that, controlling for a host of confounders, self-assessed overweight declines with rising average BMI and the obesity rate in the reference group (persons of same age and sex). In contrast, using the methodology of Glaeser et al. (2003), Auld (2011) finds only small contemporaneous social multipliers for BMI at the county and state level in the US in 1997-2002. Since the method focusses on contemporaneous interaction, it provides indirect support for a dynamic process of a gradually decreasing social disapproval of being overweight.

There exists a large literature of economic theories on obesity but social interaction is relatively neglected. Some empirical studies on obesity and social interaction are motivated with rudimentary models (Etilé, 2007, Blanchflower et al., 2008). Burke and Heiland (2007) propose a model of social dynamics of obesity, which – like the present study – emphasizes the role of a social multiplier in the gradual amplification of obesity prevalence. The solution method, however, is purely numerical; no general results are derived analytically. Wirl and

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3For a survey see Rosin (2008); see also the extensive discussion in Cutler et al. (2003), Lakdawalla et al. (2005), and Philipson and Posner (2009). The economic literature on social norms, based on Granovetter (1978) and Bernheim (1994), has already provided important insight into other phenomena, including the growing welfare state (Lindbeck, et al., 1999), out-of-wedlock childbearing (Nechyba, 2001), family size (Palivos, 2001), women’s labor force participation (Hazen and Maoz, 2002), occupational choice (Mani and Mullin, 2004), contraceptive use (Munshi and Myaux, 2006), work effort (Lindbeck and Nyberg, 2006), cooperation in prisoner’s dilemmas (Tabellini, 2008), doping in sports (Strulik, 2012), and education (Strulik, 2013a).
Feichtinger (2010) propose a mathematically more involved model in a similar spirit.\(^4\) Levy (2002) and Dragone and Savorelli (2011) propose dynamic theories of body size evolution in which a socially desirable weight is exogenous and parametrically given. In contrast, the present paper assumes a simpler static relation between food consumption and body weight and focuses on the influence of an endogenous and slowly evolving socially desirable weight on the distribution of body weight in a society.\(^5\)

The present study tries to prove as many results as possible analytically and explicitly considers idiosyncratic differences in preferences and income. Moreover, extensions of the basic model demonstrate robustness of results with respect to dietary choice and physical exercise, factors, which have not been addressed in this context so far. The present study thus proposes a theory that is suitable to explain the socio-economic gradient of obesity and to provide a comprehensive understanding of the social evolution of obesity.

2. The Model

2.1. Setup of Society. Consider a society consisting of a continuum of individuals of fixed height, which is for simplicity normalized to unity, implying that weight equals BMI. Later on, in the numerical part of the paper the normalization allows for an easy comparison of results with actual data on obesity. Individuals experience utility from food consumption and from consumption of other goods. Consumption of food of individual \(i\) in period \(t\) is denoted by \(v_t(i)\) and other consumption is denoted by \(c_t(i)\). The relative price of food is given by \(p_t\).

Each individual faces a given income \(y(i)\) and thus the budget constraint (1).

\[
y(i) = c_t(i) + p_t v_t(i).
\]

We allow income to be individual-specific but keep it constant over time in order to focus on social dynamics.

\(^4\)The focus of the study by Wirl and Feichtinger (2010) is on general social dynamics and in particular on the possibility of thresholds, i.e. unstable social equilibria that separate multiple locally stable equilibria. The application to body size is thus rather rudimentary and does not include a role for income, non-food consumption, health effects, physical exercising for weight loss, dietary choice, and the energy density of food.

\(^5\)An extension of the paper considering body weight as a slowly moving state variable is available as Strulik (2013b).
Units of food are converted into units of energy by the energy exchange rate $\epsilon$ such that individual $i$ consumes $\epsilon v_t(i)$ energy units in period $t$. For simplicity we assume that the period length is long enough – say, a month – such that we can safely ignore the specific (thermo-) dynamics of how energy consumption relates to energy expenditure and fat cell generation and growth. Instead we assume that there exists an ideal consumption of energy per period such that any consumption beyond it translates one-to-one into excess weight. Let parameter $\mu(i)$ denote lean body mass of individual $i$. The overweight status of individual $i$ in period $t$ is then given by (2).\footnote{In an accompanying technical paper (Strulik, 2013b) I develop and solve the associated dynamic model, in which body weight evolves as a state variable and consumers maximize utility over an infinite horizon. I show that all the main results developed here hold true there as well. Focussing on the simple model with a one-to-correspondence of eating and body weight and consumers with a short (one-period) planning horizon thus causes little loss of information but provides a great gain in simplicity. See, for example, Cutler et al. (2003) for a similar simplifying assumption.}

$$o_t(i) = \epsilon v_t - \mu(i) \geq 0.$$ (2)

In order to simplify the algebra we impose the constraint that $o_t(i) \geq 0$. This condition requires that all individuals eat at least as much to fulfill the metabolic needs of a lean body and allows us to avoid considering explicitly that some people in society may prefer to be underweight. The energy consumption needed to support the metabolic needs of a lean body, $\epsilon v_t(i) = \mu(i)$, can be conceptualized as “subsistence needs”. We allow $\mu(i)$ to be individual-specific because subsistence needs vary with height and occupation (muscle mass and energy exertion at work).

In the basic model, in which there exists just one type of food, all individuals face the same energy exchange rate. Section 4 sets up an extension with two types of diets for which different prices and energy exchange rates apply (junk food and healthy food). This allows us to optimize over the selection of a specific diet. It will be shown that all results from the basic model, in which individuals can only choose the quantity but not the quality of their food, hold true in the extended version as well.

Being overweight causes health costs and social costs, which are both assumed to increase in excess weight. Health costs per unit of excess weight, $\eta$, are treated parametrically, which provides an interesting experiment of comparative statics with respect to medical technological
progress. The arrival of beta blockers, for example, can be thought of as a reduction of \( \eta \). The social cost of being overweight \( s_t \), in contrast, is explicitly treated as a variable in order to address social dynamics. The presence of health costs and social costs diminishes utility from consumption. The compound of health implications and social disapproval of being overweight multiplied by the individual’s actual overweight status, given by \( (s_t + \eta)\alpha_t \), captures the negative utility resulting from the actual overweight status \( \alpha_t \). To better relate to the sociological literature, the compound can be interpreted as weight-dissatisfaction and as a proxy of the self-perception of being overweight or obese. A similar assumption has been made by Blanchflower et al. (2009).

Specifically, we assume that the utility of individual \( i \) in period \( t \) is given by

\[
U_t(i) = [c_t(i) + \beta(i)v_t(i)]^\alpha \cdot \left[ 1 - (s_t + \eta)\alpha_t(i) \right]^{1-\alpha}.
\] (3)

Here, \( \alpha \) measures the relative importance of consumption for utility compared to the consequences of food consumption on health and social disapproval, \( 0 < \alpha < 1 \). In order to arrive at an explicit solution, I have assumed that utility is non-separable and that the elasticity of substitution between food and non-food is infinite (once metabolic subsistence needs are met). Later on I show that most of the results can be also obtained when the elasticity of substitution is finite. The parameter \( \beta(i) \) measures how pleasurable food consumption is compared to other consumption. It can be thought of as a compound consisting of a common term \( \beta \) and an idiosyncratic term \( \hat{\beta}(i) \), that is \( \beta(i) \equiv \beta \cdot \hat{\beta}(i) \). Whereas \( \hat{\beta}(i) \) measures the “sweet tooth” of person \( i \), the common component \( \beta \) controls for the state of food processing technology, i.e. the general desirability of food (influenced, for example, by flavor enhancing technologies).

\[ \text{Most of the results from comparative static analysis can be obtained for general utility functions } U_t = u(c_t, v_t) \cdot x(\alpha_t) \text{ or } U_t = u(c_t, v_t) + x(\alpha_t) \text{ with } \partial u/\partial c_t > 0, \partial^2 u/\partial c_t^2 < 0, \partial u/\partial v_t > 0, \partial^2 u/\partial v_t^2 < 0, \text{ and } \partial x/\partial \alpha_t < 0, \partial^2 x/\partial \alpha_t^2 < 0, \text{ irrespective of whether the subutilities } u \text{ and } x \text{ are combined multiplicatively or additively. However, the result that poor persons are more prone to be overweight requires additional assumptions on curvature of the utility function, which are more easily fulfilled in the non-separable case. Notice furthermore that positive utility in (3) requires that } 1 > (s_t + \eta)\alpha_t, \text{ which is assumed to hold by appropriate choice of units of measurement.} \]
Individual self-control problems, although not explicitly modeled, can be thought of as being captured by the idiosyncratic preference parameter $\hat{\beta}(i)$. Persons with a dominant affective system experience more gratification from food consumption (above metabolic needs) and display a larger $\hat{\beta}(i)$ compared to more deliberative persons. A detailed understanding of how psychological mechanisms are affecting obesity is certainly useful and it has been advanced within an economic framework elsewhere (e.g. Cutler, 2003, Philipson and Posner, 2003, Ruhm, 2010). Lumping these aspects together in one compound parameter is only justified by the focus of the present study, which focusses neither on psychological nor on technological aspects but on the interaction of eating behavior and social disapproval of obesity.

2.2. Individual Utility Maximization. Any individual $i$ is assumed to maximize utility (3) subject to the budget constraint (1) and the weight constraint (2). The first order condition for an interior solution requires that

$$\alpha(\beta(i) - p_t) [y(i) + (\beta(i) - p_t)v_t(i)]^{\alpha-1} \cdot [1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i))]^{1-\alpha} = \epsilon(s_t + \eta)(1 - \alpha) [y(i) + (\beta(i) - p_t)v_t(i)]^\alpha \cdot [1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i))]^{-\alpha}. \tag{4}$$

Marginal utility from food consumption, at the left hand side of the equation, is required to equal marginal disutility from the consequences of food consumption on being overweight, at the right hand side. A necessary but not sufficient condition for excess food consumption is $\beta(i) > p_t$. To see this, notice that the terms in square brackets in (4) have to be positive for positive and concave utility and recall that $0 < \alpha < 1$. The result is intuitive. Because the price of non-food has been normalized to one, it means that for excess food consumption to occur, food consumption has to provide higher utility than non food consumption ($\beta(i) > 1$), or the price of food has to be lower than the price of non-food ($p_t < 1$), or both. Otherwise, the non-negativity constraint binds and individuals derive pleasure from eating only until their ideal metabolic needs are fulfilled, $\epsilon v_t(i) = \mu(i)$. This means that for overweight status to be an observable phenomenon, $\beta(i) > p_t$ has to hold for at least some individuals in society.$^8$

$^8$The condition thus requires that for the subutility function $u(c, v) = (c + \beta)^\alpha$ the marginal rate of substitution, $(\partial u/\partial v)/(\partial u/\partial c) = \beta$ exceeds the relative price of food $p$. This is so because the consumer takes as well in consideration the health consequences and social disapproval of overweight.
The solution of (4) provides the optimal food consumption for person $i$ in period $t$:

$$v_t(i) = \frac{\alpha}{\epsilon(s_t + \eta)} + \frac{\alpha \mu(i)}{\epsilon} - \frac{(1 - \alpha)y(i)}{\beta(i) - p_t}. \quad (5)$$

Together with the weight constraint (2) it implies that the overweight status of person $i$ is obtained as (6).

$$\omega_t(i) = \max \left\{ 0, \frac{\alpha}{s_t + \eta} - \omega(i) \right\}, \quad \omega(i) \equiv (1 - \alpha) \left( \frac{\epsilon y(i)}{\beta(i)} - p_t + \mu(i) \right). \quad (6)$$

### 2.3. Comparative Statics.

We next discuss the solution for given prices $p_t$ and social approval $s_t$. As shown in (6) excess food consumption of individual $i$ is decreasing in the degree of social disapproval of being overweight $s_t$. For any given $s_t$, inspection of (6) proves the following comparative statics.

**Proposition 1.** Consider a society defined as a probability distribution of tastes $\beta(j)$ and incomes $y(j)$ for persons $j \in N$. Then, the probability that a person $i$ is overweight is decreasing in her or his personal income $y(i)$, the unhealthiness of being overweight $\eta$, the price of food $p_t$, and the energy exchange rate $\epsilon$. It is increasing in the personal degree of gratification from food consumption $\beta(i)$ and the weight of consumption in utility $\alpha$.

**Proposition 2.** The weight of an overweight person $i$ is decreasing in income $y(i)$, the unhealthiness of being overweight $\eta$, the price of food $p_t$, and the energy exchange rate $\epsilon$. It is increasing in the personal degree of gratification from food consumption $\beta(i)$ and the weight of consumption in utility $\alpha$.

The result with respect to income helps to explain the observed negative socioeconomic gradient in obesity, that is why – ceteris paribus – richer people are less heavy. For an intuition it is useful to return to the first order condition (4). Higher income allows for a higher level of consumption, $c + \beta v$, be it in terms of food or non-food. A higher level of consumption in turn means lower marginal utility from consumption relative to the marginal disutility experienced from being overweight. It implies that the marginal utility experienced from being less heavy, measured by the right hand side (4), is higher for richer persons. In simple words, when many
consumption needs are fulfilled, health considerations and social approval of one’s appearance becomes relatively more important for individual happiness. Consequently, richer persons are, on average, less heavy. Notice that food is not an inferior good in the conventional sense. According to the subutility function \( u(c, v) \) an increase in income would induce more food consumption for \( \beta(i) > p \) and leave food consumption unaffected for \( \beta(i) \leq p \). The concerns about health and social disapproval are responsible for the negative income effect on food consumption.

The other comparative static results from Proposition 1, except for the energy exchange rate, are intuitive. The result with respect to the energy exchange rate, at first sight, appears to contradict the empirical observation that obese people are consuming particularly energy-dense food. Within the present framework, however, this seemingly counterfactual result is consistently explained: a higher energy exchange rate increases the negative consequences of excess food consumption on health and social disapproval, a fact, which discourages the incentive to eat a lot. In order to explain the empirical regularity between energy density and obesity the model has to be extended by allowing individuals to chose a particular diet. This will be done in Section 4. The seemingly counterfactual result will be resolved by allowing energy-dense diets to be either cheaper or more pleasurable or both. All other results from the simple model will be preserved.

2.4. Robustness Check: Arbitrary Elasticity of Substitution between Food and Non-Food. The assumption of perfect substitution between food and non-food has been made in order to allow for an explicit solution of the problem at hand. But how far is the simplifying assumption driving the results? In order to answer this question we re-consider the original problem now allowing for an arbitrary elasticity of substitution between food and non-food. The generalized utility function is given by

\[
U_t(i) = \{\theta [c_t(i)]^\rho + (1 - \theta) [v_t(i)]^\rho\}^{\frac{\rho}{\theta}} \cdot [1 - (s_t + \eta) o_t(i)]^{1-\alpha},
\]

with \( \rho \leq 1 \). The implied elasticity of substitution between food and non-food is \( \sigma = 1/(1-\rho) \). For \( \rho = 1 \) we obtain the simple model discussed so far \( \sigma = \infty \).
After inserting the budget constraint into the utility function the maximization problem reads

$$\max_{v_t(i)} U_t(i) = \{\theta [y(i) - p_t v_t(i)]^\rho + (1 - \theta) [v_t(i)]^\rho\}^{\frac{1}{\rho}} \cdot [1 - (s_t + \eta) o_t(i)]^{1-\alpha}.$$ 

The first order condition for optimal food consumption, after applying some algebra, reduces to

$$0 = G(v_t, \ldots) \equiv \alpha\left\{\theta [y(i) - p_t v_t(i)]^{\rho-1} \cdot (-p) + (1 - \theta) [v_t(i)]^{p-1}\right\} \cdot [1 - (s_t + \eta)(e v_t(i) - \mu(i))]$$

$$- (1 - \alpha)(s_t + \eta)\rho \theta [y(i) - p_t v_t(i)]^\rho + (1 - \theta) [v_t(i)]^\rho.$$ 

(7)

Dividing (7) by $\theta$, defining $\beta \equiv (1 - \theta)/\theta$, and setting $\rho = 1$ we get (4), confirming that the simple model is a special case of the general model when $\sigma = \infty$.

Generally, (7) has no explicit solution but the comparative statics can be assessed using the implicit function theorem (see Appendix):

**Proposition 3.** Given an arbitrary elasticity of substitution between food and non-food $\sigma$, the body weight of an overweight person is decreasing in the unhealthiness of being overweight $\eta$ and the energy exchange rate $\epsilon$ and increasing in the weight of consumption in utility $\alpha$.

For $\sigma \leq 1$ body weight is monotonically increasing in income and for $\sigma = \infty$ body weight is monotonically decreasing in income. For $1 < \sigma < \infty$ body weight is increasing in income if income is sufficiently small and decreasing in income otherwise.

The proposition is the generalized version of Proposition 2 and, because of symmetry, an analogous generalized version can be stated for Proposition 1 (here omitted for brevity). While most of the results generalize towards arbitrary elasticity of substitution, the conclusions with respect to the income-obesity nexus are qualified in an interesting way, suggesting a non-monotonic response of body weight to income for $1 < \sigma < \infty$.\(^9\) The result is helpful to explain the long-run historical trends of the income body weight association, i.e. why income and body size were positively associated for most of human history, when income was low and average income close to subsistence level, and why income and body size are negatively

\(^9\)A similar non-monotonous response of body weight to price changes can be obtained for $\sigma < 1$ (here omitted for brevity).
associated when income is relatively high, as in developed present day countries. The reason is that, for $1 < \sigma < \infty$, the effect of health and social disapproval becomes dominating only when income is high enough, i.e. when the marginal utility from consumption is low enough. The result may be employed as well to explain why some studies find for contemporaneous societies that body weight is monotonically decreasing in income for some subsamples, for example for women, but inversely u-shaped for other subsamples, for example for men (Lakdawalla and Philipson, 2009). Most importantly the result ensures that, as long as $\sigma > 1$, the simple model is a reasonable approximation of the general model if income is sufficiently large. Having made this qualifying note, we now return to the simple model.

2.5. **Social Disapproval.** Inspired by the observed social attitudes towards obesity (presented in the Introduction) we assume that social disapproval of obesity is inversely related to the actual prevalence of obesity in society. The simplest conceivable way to implement this notion is to assume that social disapproval is inversely related to the overweight status of the median person, denoted by $\bar{o}_t$. Henceforth idiosyncratic parameters that apply to the median are identified by “upper bars”, that is, for example, $o_t(i) = \bar{o}_t$ for the median.\(^{10}\)

In order to discuss social dynamics explicitly we assume that social disapproval evolves as a lagged endogenous variable depending on the observation of actual obesity in the history of the society. Let $\delta$ denote the rate of oblivion by which the historical prevalence of obesity is depreciated in mind so that disapproval is given by $s_t = (1 - \delta) \sum_{i=0}^{\infty} \delta^i g(\bar{o}_{t-i})$. Alternatively, this can be written in period-by-period notation as $s_t = \delta \cdot s_{t-1} + (1 - \delta) \cdot g(\bar{o}_t)$. Another transformation of this expression writes the change of disapproval as a function of its level, i.e. $s_t - s_{t-1} = (1 - \delta) [g(\bar{o}_t) - s_{t-1}]$ and illustrates that $\delta$ controls the adjustment speed at which social disapproval responds to changes in body weight of the median person. The lower $\delta$ the greater is the influence of the currently observable weight of the median for social disapproval of being overweight. This formulation is reminiscent of the adjustment of external disapproval of being overweight. This formulation is reminiscent of the adjustment of external

\(^{10}\)The theory is not generally based on the notion that $\bar{o}_t$ is attached to the median person. In principle social disapproval could originate from the weight of any reference individual (or reference group). Taking the median person makes it easier to focus on a society as the population of country and thus to relate to the empirical studies from the Introduction. It is also essential for the numerical exercise on evolving BMI distributions (Section 3.4). In the Conclusion I briefly discuss the possibility of different reference individuals or groups.
consumption habits (e.g. Carroll et al., 2000). In the present context it means that the median person does not internalize that his or her eating behavior has an impact on the evolution of social disapproval of being overweight.

Using the simplest conceivable inverse function \( g(x) = 1/(\gamma + x) \), social disapproval of being overweight in period \( t \) can be expressed as\(^{11}\)

\[
s_t = \delta \cdot s_{t-1} + (1 - \delta) \cdot g(\bar{o}_t), \quad g(\bar{o}_t) \equiv \frac{1}{\gamma + \bar{o}_t}. \tag{8}
\]

The parameter \( \gamma > 0 \) controls the strength of social norms. The positivity of \( \gamma \) ensures that a social equilibrium is feasible at which the median is lean and some members of society are overweight. Without \( \gamma \) social disapproval would not be defined when the median is lean (for \( \bar{o}_t = 0 \)). Moreover \( \gamma \) works as a control for the strength of social disapproval for being overweight; \( 1/\gamma \) is the maximum disapproval generated by society, that is the disapproval (per kilogram overweight) that a person experiences when the median person is lean. Notice that a lean median person does not imply that no-one is overweight in society, because individuals are heterogenous in preferences and income and some individuals, poorer than the median or with a stronger preference for eating, may be overweight even if a lean median implies a high degree of social disapproval for being big.

The important implication of (8) is that overweight individuals experience gradually less social disapproval of their appearance as the median (or reference) person gets bigger. In conjunction with the utility function it means that for any given weight individuals less strongly assess themselves as being overweight. This implication is empirically supported for US citizens by Burke et al. (2010) who compare different periods of observation from the National Health and Nutrition Examination Survey and find that self-assessments of being overweight declines with rising average BMI and the obesity rate in the reference group. A similar observation has been made for the UK by Johnson et al. (2008).

\(^{11}\)The results of comparative static analysis below do not depend on the functional specification of \( g(\bar{o}_t) \) and can be obtained for general \( g > 0 \) with \( g' < 0 \), given that the function supports an equilibrium at which the median is overweight.
3. The Social Evolution of Obesity

3.1. Steady-State. At the steady state, \( p_t = p, \ s_t = s, \) and \( \bar{o}_t = \bar{o} \) for all \( t \) and solving (8) for \( s \) provides (9).

\[
s = g(\bar{o}) \equiv \frac{1}{\gamma + \bar{o}}. \tag{9}
\]

From (6) we observe excess food consumption of the median person in period \( t \) as \( \bar{o}_t = \alpha/(s_t + \eta) - \bar{\omega} \), in which the compound parameter \( \bar{\omega} \) summarizes the impact of preferences and income of the median, \( \bar{\omega} = (1 - \alpha)\epsilon\bar{y}/(\beta - p_t) - \mu \). Solving for \( s_t \)

\[
s_t = \frac{\alpha}{\bar{o}_t + \bar{\omega}} - \eta \equiv h(\bar{o}_t). \tag{10}
\]

Diagrammatically, (9) and (10) establish two equations for social disapproval. Equation (10) holds everywhere, equation (9) holds only at the steady state, implying that the steady state fulfils both equations. Equating (9) and (10) and solving for \( \bar{o} \) provides (11).

\[
\bar{o} = o^* = -\frac{r}{2} + \sqrt{\frac{r^2}{4} - q}, \quad r \equiv \frac{1 - \alpha + \eta(\bar{\omega} + \gamma)}{\eta} > 0, \quad q \equiv \frac{\bar{\omega}(1 + \eta\gamma) - \alpha\gamma}{\eta}. \tag{11}
\]

From the fact that \( r > 0 \) it follows that there exists a unique steady state at which the median is overweight iff \( q < 0 \), that is iff \( 1/\gamma < (\alpha/\bar{\omega}) - \eta \).

The steady state and its comparative statics can be best analyzed diagrammatically. For that purpose note that both \( g(\bar{o}) \) and \( h(\bar{o}) \) are decreasing and convex in \( \bar{o} \). The graph of \( g(\bar{o}) \) originates at \( 1/\gamma \) and approaches zero as \( \bar{o} \) goes to infinity. The graph of \( h(\bar{o}) \) originates at \( \alpha/\bar{\omega} - \eta \) and approaches \( -\eta \) as \( \bar{o} \) goes to infinity. From (11) we know that there exists either no or one intersection in the positive quadrant, identifying the obesity equilibrium. These two cases are displayed in Figure 1.

If there exists no intersection of \( g(\bar{o}) \) and \( h(\bar{o}) \), as displayed on the left hand side of Figure 1, there exists no steady state of obesity as a social phenomenon. For any given perturbation resulting in the median being overweight, social disapproval \( s \) is higher than the level needed to sustain this weight as a steady state. Consequently, the median eats less until he or she returns to the corner solution where \( \bar{o} = 0 \). At the steady state the median person is not
overweight. This in turn means that, although there are overweight persons in society at the steady state (for example those poorer than the median or those with a “sweeter tooth”), being overweight is not a mass phenomenon and there exists no obesity epidemic. Any perturbation or any marginal change of parameters would induce adjustment dynamics back to $\bar{o} = 0$. There is no permanent evolution towards larger bodies in society.

The right hand side of Figure 1 shows the other, more interesting, possibility. Here the $h(\bar{o})$–curve lies above the $g(\bar{o})$ curve for small $\bar{o}$. This means that for any overweight status $\bar{o} < o^*$ social disapproval is lower than needed to sustain this weight as a steady state. Consequently, the median person (and thus any overweight person in society) eats more and puts on more weight and social disapproval of being overweight decreases until $\bar{o}$ approaches $o^*$. Excess weight above $o^*$, though, is not sustainable. The associated disapproval leads to less excess food consumption and less overweight. In other words, the equilibrium at $o^*$ is stable. The following proposition summarizes the results.
Proposition 4. There exists a stable steady state at which the median person is overweight and being overweight receives relatively little social disapproval iff

\[ \frac{1}{\gamma} < \frac{\alpha}{\overline{\omega}} - \eta. \]  

(12)

Otherwise, the median person is not overweight at the steady state and social disapproval of being overweight is relatively high.

Proposition 5. There exists a stable steady state at which the median person is overweight and being overweight receives little social disapproval if individuals care sufficiently little about the consequences of being overweight (if \( \alpha \) is sufficiently large), if being overweight entails sufficiently minor consequences on health (if \( \eta \) is sufficiently low), if the steady-state price of food is sufficiently low (\( p \) is sufficiently low), if the median person likes eating sufficiently strongly (if \( \bar{\beta} \) is sufficiently large), and if the median is sufficiently poor (\( \bar{y} \) is sufficiently low).

The proof evaluates \( \omega(i) \) in (6) for the median and inserts the result into (12) which provides the condition

\[ \frac{1}{\gamma} + \eta < \frac{\alpha}{1 - \alpha} \cdot \frac{\bar{\beta} - p}{\bar{\epsilon} \bar{y} - (\bar{\beta} - p)\bar{\mu}}. \]  

(13)

Inspection of (13) verifies the proposition.

Using Proposition 1 and 2 and inspecting Figure 1 it is straightforward to derive the comparative statics of the social equilibrium. For that purpose it is helpful to note that the \( g(\bar{o}) \)-curve remains unaffected by value changes of the parameters \( \alpha, \bar{\beta}, p, \bar{y}, \bar{\mu}, \) and \( \eta \). The fact that Proposition 2 holds true for any person (and thus in particular for the median person) and at any \( s_t \) (and thus in particular at the steady state) implies that comparative statics for these parameters can be obtained simply be observing how they shift the \( h(\bar{o}) \)-curve. Applying Proposition 2 we see that increasing \( \alpha, \bar{\beta} \) and decreasing \( \eta, p, \) and \( \bar{y} \) shift the \( h(\bar{o}) \)-curve to the right, in direction of heavier bodies. This observation proves the following proposition.

Proposition 6. If a social equilibrium of obesity \( o^* \) exists, then the median person is heavier and the prevalence of being overweight in society is higher if individuals care less
about the consequences of eating (if $\alpha$ is larger), if the median has a greater preference for eating (if $\tilde{\beta}$ is larger), if health consequences of overeating are smaller (if $\eta$ is smaller), if the price of food $p_t$ is lower, and if the median person is poorer (if $\bar{y}$ is smaller).

The last result provides a rationale for why, apparently, obesity is more prevalent in unequal societies (see the Introduction). Controlling for average income the median is poorer in unequal societies and – due to the mechanism explained above – motivated to eat more. This implies that being overweight attracts less social disapproval and that other members of society are (more severely) overweight as well.

For the comparative static result on $\gamma$, note that the size of $\gamma$ affects only the $g(\bar{o})$–curve but not the $h(\bar{o})$–curve. A higher $\gamma$ shifts the $g(\bar{o})$ curve downwards. This means that, if a social equilibrium of obesity $o^*$ exists, the median is heavier and the prevalence of being overweight in society is higher if being overweight is less punished with disapproval by society.

Shifts of parameters that apply to all persons have a two-fold consequence on individual body size. There is a social multiplier at work. We next consider two examples for the multiplier with interesting and perhaps non-obvious results.

3.2. Feeling Unhealthy. If medical technological progress (e.g. the arrival of beta blockers, dialysis, coronary stents) reduces the health consequences of being overweight, some persons are motivated to eat more. If the median is among these persons, which is the case when $o^*$ exists, there is a social multiplier at work. Formally we can define unhealthiness as the part $\eta \cdot o_t \equiv u(o_t)$ in utility. Evaluating this expression for the median at the steady state and taking the first derivative with respect to $\eta$ we get:

$$\frac{\partial u}{\partial \eta} = \bar{o} + \eta \cdot \frac{\partial \bar{o}}{\partial \eta}.$$  \hspace{1cm} (14)

The first term in (14) identifies the direct effect of the health innovation on health of the median. It is positive. For decreasing $\eta$, representing medical technological progress, this means that the median feels less unhealthy. This fact, however, motivates her (and thus a majority of society) to eat more and to put on more weight. The second term in (14) identifies
the negative consequences of increasing body weight on health through the social multiplier. It is negative (recall Proposition 2).

Figure 2: Medical Technology, Obesity, and Health of the Median Person

Evaluated at steady state. Lower values of \( \eta \) are associated with a higher level of medical technology, i.e. technology improves from left to right. Body mass index (BMI) is given by \( \bar{\mu} + \bar{o} \). Unhealthiness is measured by \( u(\bar{o}) = \eta \bar{o} \). Parameters: \( p = 1, \alpha = 0.8, \beta = 2, \gamma = 50, \epsilon = 2.5, \bar{\mu} = 23, \bar{y} = 10. \)

Due to the counteracting forces on the response of unhealthiness, it can happen that the social effect dominates the individual effect such that the median (and other members of society) are less healthy at the new steady state after a positive innovation of health technology. Figure 2 verifies this claim by way of example. It shows the steady-state value of weight and the experienced unhealthiness by the median for alternative levels of medical technology. Without excess eating the parameterized median would have a lean body mass index \( \bar{\mu} \) of 23. Values for the other parameters are given below Figure 2. Notice that the abscissa is scaled by \( 1 - \eta \) such that movements to the right represent improvements of medical technology. Coming from a low level of obesity-related health technology, that is from the left (high \( \eta \)), a situation, which is associated with a mildly overweight median person, the social multiplier causes the median to be heavier and unhealthier at the steady state when \( \eta \) decreases. This means that unhealthiness \( u(\bar{o}) \) is increasing with medical technological progress. Only if the
state of medical technology is very high, the \( u(\bar{o}) \) curve is negatively sloped, implying that further improving technology leads to less severe health consequences at the steady state.

3.3. Feeling Fat. A similar consideration can be made for the impact of social attitudes on self-perception. The impact of social disapproval on the experienced disutility from being overweight is measured by the degree of “feeling fat” \( f(o_t) \equiv s_t o_t \). Evaluating the expression for the median at the steady state,

\[
f(\bar{o}) = \frac{\bar{o}}{\gamma + \bar{o}},
\]

and taking the derivative with respect to \( \gamma \) we obtain (15).

\[
\frac{\partial f(\bar{o})}{\partial \gamma} = -\frac{1}{(\gamma + \bar{o})^2} \cdot \bar{o} + \frac{\gamma}{(\gamma + \bar{o})^2} \cdot \frac{\partial \bar{o}}{\partial \gamma}.
\]

(15)

The first term identifies again the direct effect and it is negative. When \( \gamma \) rises, individuals experience less social disapproval at any given body size. The median (and other persons in society) are feeling less fat. At a steady state of obesity \( o^* \), however, this fact motivates eating more and putting on more weight. The negative effect of the social multiplier on disapproval is measured by the second, positive term. Individuals “feel fatter” due to the weight gain.

Again, it can happen that the social effect dominates the direct effect. Another example, shown in Figure 3, corroborates this claim. It shows the steady-state weight and the experienced utility loss from social disapproval for alternative values of \( \gamma \). When disapproval for being overweight is very low (1/\( \gamma \) is low), the median person is very fat at the steady state but suffers relatively little from the evaluation of others. Many other persons are anyway obese themselves. At the other extreme, when being overweight is severely punished with disapproval, the median is only mildly overweight and suffers mildly from “feeling fat”. At an intermediate degree of social disapproval and an intermediate degree of overweight the suffering from social disapproval is largest. In other words, coming from the right from a situation of high social disapproval of overweight (high 1/\( \gamma \)), less disapproval per unit of overweight leads to heavier persons and actually to more suffering from social disapproval.
Evaluated at steady state. Lower values of $1/\gamma$ are associated with lower social disapproval of being overweight. Body mass index (BMI) is given by $\mu + \bar{o}$. The degree of “feeling fat” is given by $f(\bar{o}) = \bar{o}/(\gamma + \bar{o})$. Parameters as for Figure 2 and $\eta = 0.1$.

3.4. BMI Distribution. At a steady state of obesity $\sigma^*$ any overweight person responds in the same direction as the median to changes of common parameters (recall Proposition 2). Quantitatively, however, individuals can respond quite differently. To see that, take the difference of body weight for any two persons, $i = j, k$ at the steady state. From (5) we get

$$w(j) - w(k) = \alpha [\mu(j) - \mu(k)] - (1 - \alpha)\epsilon \left[ \frac{y(j)}{\beta(i) - p} - \frac{y(k)}{\beta(k) - p} \right].$$

(16)

The result shows that a change of almost every parameter changes the relative position of individuals in the weight distribution. A comprehensive discussion of the effect of innovations on overweight of all persons would thus require complete specification of a distribution of preferences and incomes.

But inspection of (16) also shows that value changes of $\eta$ and $\gamma$ do not affect the differential $w(j) - w(k)$. Since this is true for any $j$ and $k$, it means that changes of these parameters do not change the variance of any distribution of body weight. The effect of a change of $\eta$ or $\gamma$ on body weight can thus be discussed conveniently not only with respect to the median person but with respect to the distribution of body weight of the whole society.
We exploit this fact with a numerical experiment and begin with the empirical observation that body weight $w$ is approximately log-normally distributed, i.e. the partial distribution function for body weight is given by

$$f(w) = \frac{1}{w\sqrt{2\pi\nu}} \cdot e^{-\frac{(\log w-x)^2}{2\nu}}.$$ 

Recall that $x$ and $\nu$ are the mean and the variance of $\log(w)$. The median of the log-normally distributed $w$ is given by $e^x$ and thus $x = \log(\bar{w})$. The variance of $w$ is given by $\text{var}(w) = (e^\nu - 1) \cdot e^{2x+\nu}$. The fact that the variance of $w$ does not respond to changes of $\eta$ or $\gamma$ can be utilized by solving $\text{var}(w)$ for the shift-parameter $\nu$:

$$\nu = \log \left[ \frac{1}{2} + \frac{e^{-x}\sqrt{4\text{var}(w)} + e^{2x}}{2} \right] = \log \left[ \frac{1}{2} + \frac{\frac{1}{\bar{w}}\sqrt{4\text{var}(w)} + \bar{w}^2}{2} \right].$$

The result implies that the shift parameter of the log-normal can be inferred from the weight of the median person. This means that in order to study the impact of changing health consequences $\eta$ or changing social disapproval of being overweight $\gamma$ on the whole distribution of body weights one needs only to know the variance of the initial distribution of body weight and how $\eta$ or $\gamma$ affect body weight of the median person. Yet, the latter is already known from Proposition 2. An evolving body weight distribution can thus be conveniently derived by calibrating the initial variance and then feeding in new values for health technology ($\eta$) or for the strength of social disapproval of being overweight ($\gamma$).

An application of this result is shown in Figure 4. The black curve in Figure 4 shows the density function of a log-normal distribution such that it approximates the actually observed density function in 1971-75 (see Cutler et al., 2003, and Veerman et al., 2007). In particular, we have assumed that $\eta = 0.1$ and adjusted $\nu$ to 0.03 in order to fit the empirical observation. The blue (dashed) line shows the resulting weight distribution after a medical innovation had lowered the consequences of obesity. Specifically, $\eta$ had been reduced by a quarter to 0.075. The BMI distribution shifts to the right and the right hand tail gets fatter. The result roughly approximates the actual movement documented by Cutler et al. and Veerman et al.
The red (dash-dotted) line represents the prediction of the obesity distribution after a further reduction of $\eta$ to 0.05.

3.5. Adjustment Dynamics: The Evolution of Obesity. Innovations in medical technology can explain the actual evolution of the BMI distribution only imperfectly. The last decades have seen other, potentially more important body-size affecting changes. In particular, a falling relative price of food has been proposed in the literature. Some researchers, however, are confused by the observation that the major decrease of food prices occurred in the 1970s and 1980s while body weight continues to grow nowadays (Cutler et al., 2003, Ruhm, 2010). The present model is helpful in resolving the puzzle. With a social multiplier at work it can well be that most of the increase of body size occurs long after food prices declined. In other words, lower food prices initiated the rise of body weight, but it is the social multiplier that developed it further and amplified it such that the phenomenon evolved towards an “obesity epidemic” which affects a majority of society.
We next demonstrate the social evolution of obesity and the power of the social multiplier with a numerical example. For that purpose we set weight of the non-overweight median $\mu$ to 24 kg/meter$^2$. We think of the period length as of one quarter and set $\delta$ to a relatively high value of 0.9. This means that past observations of weight status in society play a relatively large role in the determination of social disapproval for being overweight, which is only gradually updated on the basis of currently observable weight status. As a result we observe a high persistence of BMI and a slow evolution of the social evaluation of appearances. Slowly evolving social disapproval is an assumption that is helpful to rationalize the actually observed evolution of body sizes over the last decades. It is consistent with Auld’s (2011) observation that the contemporaneous social multiplier is small. The full set of parameter values is specified below Figure 5.

For the initial price of food, $p(0) = 1$, the system is situated in the lean steady state. Small perturbations and small changes of parameters do not affect the existence of the steady state. Driven by social disapproval the median always returns to lean body mass and the BMI distribution in society is time-invariant. This setup approximates the historical situation in the US and many other developed countries before the 1980s. The experiment shown in Figure 5 assumes that, starting in such a situation, the price of food drops by 5 percent per quarter for 12 quarters. The solid line in the right panel shows the initiated evolution of median BMI. During the first half of the period of declining food prices, body weight of the median stays constant; the system is still associated with the lean steady state. Only after food prices have been falling long enough, the lean steady state becomes non-existent and the median person puts on weight and social disapproval of being overweight deteriorates.

After 12 quarters the price stops declining but the social multiplier continues to operate. The new steady state is not yet reached. Actually, we observe that median weight in society increases by more during the period of constant prices than during the period of declining prices. Because social disapproval of being overweight continues to decline, the median (and thus society at large) continues to eat more, which in turn further reduces social disapproval etc. An observer unaware of the underlying social dynamics might thus wrongly conclude
that falling food prices cannot have caused the obesity epidemics. A similar argument can be made with respect to the preference parameter $\bar{\beta}$, which affects weight status inversely to $p$ (see equation (6)). Technological innovations which improved the palatability (flavor enhancer) or availability (convenience food) of food and therewith increased its likeability, measured by an increase of $\bar{\beta}$, can have initiated an obesity evolution, which becomes only fully visible long after the innovation took place.

The dashed (green) and dash-dotted (red) lines in Figure 5 reflect the associated BMI evolution for individuals which are 20 percent richer or poorer, respectively, than the median but face otherwise identical preferences and technologies. While the poor individual reacts immediately on falling food prices, the rich individual keeps a lean body as long as prices are falling (caused by the yet high social disapproval for a non-lean appearance) and starts putting on weight only after food prices have settled down. One could thus argue that overeating by the rich individual was not motivated by falling prices but by declining social disapproval. This view, however, fails to acknowledge that falling food prices and the triggered median behavior have caused social disapproval to decline sufficiently such that overeating became attractive for the rich individual.
4. Extensions

4.1. Physical Exercise and Weight Loss. In this section we investigate robustness of results when individuals have the possibility to lose weight through physical exercise. An individual \( i \) who decides to spend \( e_t(i) \) units of leisure on physical exercises, gets rid of \( \lambda e_t(i) \) units of body weight, that is \( o_t(i) = \epsilon v_t(i) - \mu(i) - \lambda e_t(i) \). The parameter \( \lambda \) controls how effective exercising is with respect to weight loss. The opportunity cost of exercising is that less leisure time is available for other activities. Keeping the multiplicative form of utility in order to allow for an explicit solution we assume that exercising reduces utility by factor \((1 + e_t(i))^{-\phi(i)}\). The parameter \( \phi(i) \) controls how much the person dislikes physical exercise, \( 0 < \phi < 1 \). This formulation allows the simple model from Section 2 as the corner solution when the individual does not exercise \((e_t = 0)\). In the following we focus, however, on the interior solution. Specifically, utility of person \( i \) is rewritten as

\[
u_t(i) = \left[c_t(i) + \beta(i)v_t(i)\right]^\alpha \cdot [1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i) - \lambda e_t(i))]^{1-\alpha} \cdot [1 + e_t(i)]^{-\phi(i)}.
\] (17)

The first order condition with respect to food consumption and exercise can be solved for the interior solution (18) and (19). They imply overweight status (20).

\[
v_t(i) = \frac{\alpha (\beta(i) - p_t) \left[1 + (s_t + \eta)(\mu(i) - \lambda)\right] - \epsilon (1 - \alpha + \phi)(s_t + \eta)y}{\epsilon (\beta(i) - p_t)(s + \eta)(1 - \phi(i))}
\] (18)

\[
e_t(i) = \frac{(\beta(i) - p_t) \left[\phi(i) + (\mu(i) - \lambda)(s_t + \eta)\right] + \epsilon \phi(i)(s_t + \eta)y}{\lambda (\beta(i) - p_t)(s + \eta)(1 - \phi(i))}
\] (19)

\[
o_t(i) = \frac{(\beta(i) - p_t) \left\{\alpha \left[1 + (\mu(i) - \lambda)(s_t + \eta)\right] - \lambda e_t(i) + (\mu(i) - \lambda)(s_t + \eta)\right\} - (1 - \alpha)\epsilon (s_t + \eta)y}{(\beta(i) - p_t)(s + \eta)(1 - \phi(i))}
\] (20)

The solutions for \( v_t(i) \) and \( o_t(i) \) look structurally similar to those of the simple model. But there are also interesting differences. Taking the derivatives with respect to income provides:

\[
\frac{\partial e_t(i)}{\partial y_t(i)} = \frac{\epsilon \phi(i)}{\lambda D} > 0, \quad \frac{\partial v_t(i)}{\partial y_t(i)} = -\frac{1 - \alpha - \phi(i)}{D}, \quad \frac{\partial o_t(i)}{\partial y_t(i)} = -\frac{(1 - \alpha)\epsilon}{D} < 0,
\]

\[D \equiv (\beta(i) - p_t)(1 - \phi).\]
Recalling that $\beta(i) > p$ is necessary for being overweight and observing the sign of the derivatives proves the following proposition.

**Proposition 7.** *Ceteris paribus, individuals with higher income exercise more for weight loss and are less overweight. They eat less if $\phi(i) < 1 - \alpha$.*

The possibility of getting rid of weight through exercising breaks the causal link from food consumption to being overweight. Only if the impact of body size for utility is sufficiently large ($1 - \alpha$ is sufficiently large), richer people eat less. Otherwise they eat more and work out the weight gain through increased exercising. In any case, however, the original result that richer individuals are, ceteris paribus, less overweight is preserved. In line with the empirical observation the extended model predicts that richer people, on average, exercise more for weight loss (Gidlow et al., 2006).

For social dynamics the impact of $s_t$ on body weight is of particular interest. Taking the derivatives we obtain:

$$
\frac{\partial e_t(i)}{\partial s_t} = -\frac{\phi(i)}{\lambda \hat{D}} < 0, \quad \frac{\partial v_t(i)}{\partial s_t} = -\frac{\alpha}{\epsilon \hat{D}} < 0, \quad \frac{\partial o_t(i)}{\partial s_t} = -\frac{(\alpha - \phi(i))}{\hat{D}}.
$$

As for the simple model, individuals react to increasing social disapproval of being overweight by eating less. Maybe surprisingly, they also exercise less. This is so because higher disapproval $s_t$ increases the marginal utility from exercising. In order to equalize marginal utility and marginal cost, individuals reduce $e_t$. As a result the response of overweight status to $s_t$ is generally ambiguous. In order to preserve the mechanism and results from the simple model, we have to assume that the median person likes consuming sufficiently strongly, $\alpha > \bar{\phi}$, that is that he regards consuming more important than not exercising. This restriction appears to be rather mild.

### 4.2. Diet Selection, Energy-Density, and Obesity

In this section we explore one possible explanation of the positive association between energy-density and obesity. For that
purpose we extend the model such that there are two food goods. The unhealthy good, identified by index $u$, is relatively cheap, energy dense, and potentially tasty (junk food). The second good is relatively expensive, light, and potentially less palatable. Since the expressions become rather long we omit the time index and the index $i$ for idiosyncratic variables whenever this does not lead to confusion. Specifically we assume that

$$(\beta_u - p_u) > (\beta_h - p_h), \quad \epsilon_u > \epsilon_h.$$  

Good $u$ is cheaper or more preferable or both compared to good $h$ and its energy exchange rate is higher. Let $v_u$ and $v_h$ denote consumption of good $u$ and good $h$. The budget constraint and weight constraint are then given by

$$y = c + p_u v_u + p_h v_h, \quad (21)$$

$$o = \epsilon_u v_u + \epsilon_h v_h - \mu, \quad (22)$$

Furthermore, we allow consumption of good $u$ to be unhealthy beyond its impact on weight (for example because of high content of sugar or trans-fats) and measure the health effect by the parameter $\psi$. Using this fact and (21) and (22), utility (3) can be restated as

$$U = [y + (\beta_u - p_u)v_u + (\beta_h - p_h)v_h]^\alpha \cdot [1 - (s + \eta)(\epsilon_u v_u + \epsilon_h v_h - \mu) - \psi v_u]^{1-\alpha}. \quad (23)$$

Individuals are maximizing utility by choosing $v_u \geq 0$ and $v_h \geq 0$. The double linearity in (21) and (22) implies that only corner solutions are optimal. Individuals either chose the healthy diet or the unhealthy diet.\footnote{To square the corner solution with reality it is helpful to conceptualize the diets as bundles of foods and $h$ as the on average more healthy diet, which may include an occasional donut.} In the Appendix it is shown that the solution is either $v_u$ or $v_h$:

$$v_u = \frac{\alpha(\beta_u - p_u) + (s + \eta)[\alpha(\beta_u - p_u)\mu - (1 - \alpha)\epsilon_u y] - \psi(1 - \alpha)y}{(\beta_u - p_u)[\epsilon_u(s + \eta) + \psi]}, \quad (24)$$

$$v_h = \frac{\alpha(\beta_h - p_h) + (s + \eta)[\alpha(\beta_h - p_h)\mu - (1 - \alpha)\epsilon_h y]}{(\beta_h - p_h)[\epsilon_h(s + \eta)]}.$$

\footnote{To square the corner solution with reality it is helpful to conceptualize the diets as bundles of foods and $h$ as the on average more healthy diet, which may include an occasional donut.}
Inspecting (25) and (5) let us conclude that the solution for the healthy diet \( v_h \) is isomorph to the solution of the simple one-diet model. The interesting case is thus when at least some individuals prefer the unhealthy diet. Their overweight status is then given by \( o_u = \epsilon_u v_u - \mu \), that is by

\[
o_u = \frac{\alpha \epsilon_u (\beta_u - p) - (1 - \alpha) [\epsilon_u (s + \eta) + \psi] \epsilon_u y + \alpha \epsilon_u (s + \eta)(b_u - p_u)\mu}{(\beta_u - p_u) [\epsilon_u (s + \eta) + \psi]} - \mu.
\]

Inspecting the response of being overweight on energy-density provides the following result.

**Proposition 8.** Consider a person who prefers the unhealthy diet and is overweight. Then, an increase in the energy exchange rate \( \epsilon_u \) results in even more weight gain for any given level of social approval \( s \) if the unhealthy food is sufficiently cheap (\( p_u \) sufficiently low) or sufficiently tasty (\( \beta_u \) sufficiently large) or if the person is sufficiently poor (\( y \) is sufficiently low).

The proof evaluates the first order derivative

\[
\frac{\partial o_u}{\partial \epsilon_u} = \frac{\alpha (\beta_u - p_u) \psi [1 + (s + \eta)\mu] - (1 - \alpha) [\psi + \epsilon_u (s + \eta)]^2 y}{(\beta_u - p_u) [\psi + \epsilon_u (s + \eta)]^2}
\]

and the second order derivatives

\[
\frac{\partial^2 o}{\partial \epsilon_u \partial y} = -\frac{1 - \alpha}{\beta_u - p_u} < 0, \quad \frac{\partial^2 o}{\partial \epsilon_u \partial (\beta_u - p_u)} = \frac{1 - \alpha}{(\beta_u - p_u)^2} > 0.
\]

Observing that one can always find a \( (\beta_u - p_u) \) high enough and an \( y \) low enough such that \( \partial o_u / \partial \epsilon_u > 0 \) completes the proof.

For social dynamics and steady states it now matters whether the median person prefers the healthy or the unhealthy diet. Naturally, in case of a healthy diet all results from the simple model carry over to the two-diet model, because the solution for the median is isomorph. If the median person prefers the unhealthy diet, results are generally ambiguous. The response of being overweight on social disapproval is obtained as

\[
\frac{\partial o_u}{\partial s} = -\frac{\alpha \epsilon_u (\epsilon_u - \psi \mu)}{[\epsilon_u (s + \eta) + \psi]^2}
\]
Increasing social disapproval of being overweight evokes the normal response of weight loss if the constraint $\epsilon_u > \psi \mu$ holds. This means the energy density of the unhealthy good must be sufficiently high. In this case, as well as generally if the median person picks the healthy diet, all results from the basic model carry over to the two-diet-model.

Comparing the dietary choices (24) and (25) shows that one can always find a triple $\{\beta_u, \beta_h, y\}$ for which the unhealthy diet is strictly preferred. Since consumption under both diets is strictly decreasing in income, body weight in a society which is stratified only by income is distributed as follows. The poorest individuals indulge the cheap unhealthy diet and are potentially overweight. At some level of income, $v_u \geq 0$ becomes binding with equality and the richer individuals enjoy the healthy diet. While they are potentially overweight as well, eventually, as income rises further, the metabolic constraint $\epsilon_h v_h - \mu \geq 0$ becomes binding with equality. The richest individuals – due to the mechanism explained in Section 2 – refrain from excess food consumption and are not overweight.

Living on the unhealthy diet, however, does not necessarily make people fatter. To see this, consider a society stratified only by food preferences, $\beta_u$ and $\beta_h$, and focus on the limiting case, in which diet $u$ is not unhealthy aside from its energy density, that is $\psi = 0$. Holding income $y$ (and lean body size $\mu$) constant, computing $o_h(v_h) = \epsilon_h v_h - \mu$ from (25) and subtracting it from (24) provides the body size differential

$$o(v_u) - o(v_h) = \frac{(\beta_u - p_u)\epsilon_h - (\beta_h - p_h)\epsilon_u}{(\beta_u - p_u)(\beta_h - p_h)} \cdot (1 - \alpha) y.$$ 

The unhealthy eaters are thus only bigger if

$$\frac{\beta_u - p_u}{\beta_h - p_h} > \frac{\epsilon_u}{\epsilon_h}.$$ 

That is, only if eating the unhealthy food provides sufficiently great pleasure or if it is sufficiently cheap compared to its energy exchange rate and relatively to the healthy good, are the unhealthy eaters more overweight.
5. Final Remarks

This paper has proposed a theory of the social evolution of being overweight which explains the changing human phenotype since the 1980s. A social multiplier rationalizes why declining food prices or technological innovations could have initiated an obesity epidemic although the most dramatic weight gain is observed long after the initiating innovation is gone. The social multiplier can also explain how obesity-related health innovations may have detrimental steady-state effects on health and why unequal societies are, ceteris paribus, heavier. Within societies the theory explains the socio-economic gradient, i.e. why poorer people are more severely afflicted by obesity although eating is costly. Extensions have shown that the basic mechanism is robust against the consideration of dietary choice and exercising for weight loss. The extensions have furthermore provided theoretical support for the observation that exercising is more popular among richer individuals as well as a condition under which an increasing energy density of food may have caused and/or aggravated the obesity dynamic, namely if the median person indulges an unhealthy diet and is sufficiently poor.

The theory has been based on the assumption that social disapproval is influenced by the overweight status of the median person. While it seems intuitively reasonable, that weight of the average person is an important determinant, nothing hinges on this assumption. In particular we could have assumed a “role model” less heavy than the median without qualitative impact on results. A more comprehensive assumption would certainly allow the social norm to depend on the whole BMI distribution. This assumption, however, would severely complicate the analysis and has been abandoned in favor of analytically provable results. The median has been imagined (implicitly) as the median of a country since most empirical studies are carried out at the country level or across countries. But in terms of theory, the type of the investigated society is actually undetermined. It is easily conceivable that the theory of obesity evolution applies to smaller societies than countries, that is at the level of local neighborhoods or among peers at school or at work.
APPENDIX

5.1. Proof of Proposition 3. The derivative of (7) with respect to food consumption is given by

\[ \frac{\partial G}{\partial v_t(i)} = \alpha(\rho - 1) \left[ \theta(y(i) - pv_t(i))^{\rho - 2} p^2 + (1 - \theta)v_t(i)^{\rho - 2} \right] \cdot \left[ 1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i)) \right] \\
- \alpha(s_t + \eta) \epsilon \left[ \theta(y(i) - pv_t(i))^{\rho - 1} \cdot (-p) + (1 - \theta)v_t(i)^{\rho - 1} \right] \\
- (1 - \alpha)(s_t + \eta) \epsilon \rho \left[ \theta(y(i) - pv_t(i))^{\rho - 1} \cdot (-p) + (1 - \theta)v_t(i)^{\rho - 1} \right] \]

Which can be rewritten as

\[ \frac{\partial G}{\partial v_t(i)} = \alpha(\rho - 1) \left[ \theta(y(i) - pv_t(i))^{\rho - 2} p^2 + (1 - \theta)v_t(i)^{\rho - 2} \right] \cdot \left[ 1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i)) \right] \\
- [\rho + \alpha(1 - \rho)] \cdot (s_t + \eta) \epsilon \left[ \theta(y(i) - pv_t(i))^{\rho - 1} \cdot (-p) + (1 - \theta)v_t(i)^{\rho - 1} \right] < 0 \]

because \( \rho \leq 1 \) and because \( \theta(y(i) - pv_t(i))^{\rho - 1} \cdot (-p) + (1 - \theta)v_t(i)^{\rho - 1} > 0 \) for a solution of \( G(v_t(i)) = 0 \) according to (7) to exist.

The signs of the derivatives \( \partial G/\partial \alpha > 0, \partial G/\partial \eta < 0, \partial G/\partial \epsilon < 0 \) are obvious from (7). The derivative with respect to income is obtained as

\[ \frac{\partial G}{\partial y(i)} = -p(\rho - 1) \alpha \theta (y(i) - pv_t(i))^{\rho - 2} \cdot \left[ 1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i)) \right] \\
- (1 - \alpha)(s_t + \eta) \epsilon \theta \rho (y(i) - pv_t(i))^{\rho - 1} \]

The derivative is positive for \( \rho \leq 0 \) and negative for \( \rho = 1 \). It is ambiguous for \( 0 < \rho < 1 \).

To see this clearly divide the expression by \( (y(i) - pv_t(i))^{\rho - 2} \) and conclude that for \( 0 < \rho < 1 \) the sign of \( \partial G/\partial y(i) \) is the same as the sign of \( a - b \),

\[ a \equiv p(1 - \rho) \alpha \theta \cdot [1 - (s_t + \eta)(\epsilon v_t(i) - \mu(i))] > 0, \quad b \equiv (1 - \alpha)(s_t + \eta) \epsilon \theta \rho (y(i) - pv_t(i)) > 0. \]

While \( a \) is independent from income, \( b \) is zero if all income is spent on food, i.e. for \( g(i) = pv_t(i) \), and otherwise linearly increasing in income. Hence the derivative is positive for low income and negative for high income.

Next use from the implicit function theorem, \( dv_t(i)/dx = -\frac{\partial G/\partial x}{\partial G/\partial v_t(i)} \), in which \( x \in \{ \alpha, \eta, \epsilon, y(i) \} \) and notice that \( \alpha v_t(i) / \epsilon v_t(i) \) to complete the proof of the proposition.

5.2. Derivation of (24) and (25). The Kuhn-Tucker conditions for a maximum of (23) can be simplified to

\[ v_1 U_1 = 0, \quad U_1 \equiv \alpha(\beta_1 - p_1) \left[ 1 - (s + \eta)(\epsilon_1 v_1 + \epsilon_2 v_2 - \mu) - \psi v_1 \right] \]

\[ = (1 - \alpha) \left[ (s + \eta) \epsilon_1 + \psi \right] \cdot [y + (\beta_1 - p_1)v_1 + (\beta_2 - p_2)v_2] \]

\[ v_2 U_2 = 0, \quad U_2 \equiv \alpha(\beta_2 - p_2) \left[ 1 - (s + \eta)(\epsilon_1 v_1 + \epsilon_2 v_2 - \mu) - \psi v_1 \right] \]

\[ = (1 - \alpha)(s + \eta) \epsilon_2 \cdot [y + (\beta_1 - p_1)v_1 + (\beta_2 - p_2)v_2] . \]
Suppose that both $v_1 > 0$ and $v_2 > 0$. Solving the Kuhn-Tucker conditions for $v_1$ and $v_2$ provides:

$$v_1 = \frac{-N_1}{D}, \quad v_2 = \frac{N_2}{D},$$

$$N_1 \equiv (\beta_1 - p_1)[1 + \mu(s + \eta)] + (s + \eta)\epsilon_1 y + \psi y > 0$$

$$N_2 \equiv (\beta_2 - p_1)[1 + \mu(s + \eta)] + (s + \eta)\epsilon_2 y > 0$$

$$D \equiv (\beta_2 - p_2)[(s + \eta)\epsilon_1 + \psi] - (\beta_2 - p_2)\epsilon_2(s + \eta).$$

Since both $N_1$ and $N_2$ are positive, $\text{sgn}(v_1) = -\text{sgn}(v_2)$, a contradiction to the initial claim that both $v_1$ and $v_2$ are positive. Thus, either $v_1 = 0$ or $v_2 = 0$. Solving (A.1) for $v_2 = 0$ provides (24) and solving (A.2) for $v_1 = 0$ provides (25).
References


WHO (2011), Obesity and overweight, Factsheet No. 311 (http://www.who.int/mediacentre/factsheets/fs311/en/)